Study Guide
for
Test Technician Test

Test Number: 2774

Human Resources
Southern California Edison Company

Introduction

The test is a job knowledge test designed to cover the major knowledge areas necessary to perform the job. This Guide contains strategies to use for taking tests and a study outline, which includes knowledge categories, major job activities, and study references.

Test Session

It is important that you follow the directions of the Test Administrator exactly. If you have any questions about the testing session, be sure to ask the Test Administrator before the testing begins. During testing, you may NOT leave the room, talk, smoke, eat, or drink. Since some tests take several hours, you should consider these factors before the test begins.

All cellular/mobile phones, pagers or other electronic equipment will NOT be allowed in the testing area.

All questions on this test are multiple-choice or hot spot questions. Multiple choice questions have four possible answers. Hot spot questions have a picture, and you must click the correct spot on the picture to answer the question. All knowledge tests will be taken on the computer. For more information on this, please see the next section of this study guide on Computer Based Testing.

The test has a 3 hour time limit.

A scientific calculator will be provided for you to use during the test. You will be given the choice between the following calculators: Casio fx-115es plus or Texas Instruments TI-36X.

You will NOT be able to bring or use your own calculator during testing.

You will receive a Test Comment form so that you can make comments about test questions. Write any comments you have and turn it in with your test when you are done.

Study Guide Feedback

At the end of this Guide you have been provided with a Study Guide Feedback page. If a procedure or policy has changed, making any part of this Guide incorrect, your feedback would be appreciated so that corrections can be made.

Computer Based Testing

Taking an SCE knowledge test on the computer is simple. You do not need any computer experience or typing skills. You will only use the keyboard to enter your candidate ID and password. You'll answer all questions by pressing a single button on the mouse.

Log in Screen

You will be seated at a testing station. When you are seated, the computer will prompt you to enter the candidate ID and password you received in your invitation e-mail. You **MUST** have your candidate ID and password or you will be unable to take the test. Once you have confirmed your identity by entering this information, you will see a list of tests available to you.

Sample/Tutorial

Before you start your actual test, a Sample/Tutorial Test is provided to help you become familiar with the computer and the mouse. From the list of exams that appear when you complete the log in, you will select Sample/Tutorial. You will have up to 10 minutes to take the Sample/Tutorial Test. The time you spend on this Sample Test does NOT count toward your examination time. Sample questions are included so that you may practice answering questions. In the Sample/Tutorial Test, you will get feedback on your answers. You will not receive feedback on your actual test.

Example

During the test, you may see several different types of items. Many of the questions will be multiple choice items. A few items will be pictures, where you'll have to click the spot on the picture that answers the question. Those picture questions are known as "Hot Spot" questions. More information on each type is below.

<u>Overall Test Information</u>

When you begin the test, you can see the total time allowed for completion displayed at the top of the screen. You can scroll up to see that information at any time during the test.

You can change your answers at any time during the test until the time runs out, or you click the "Submit" button. Once you click Submit, you cannot change your answers.

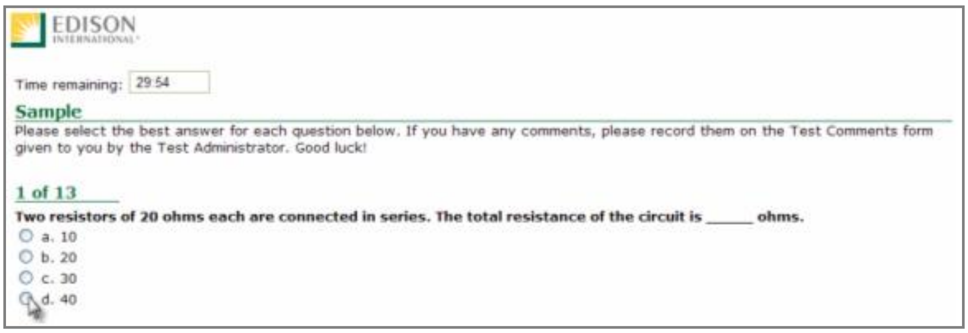
SOUTHERN CALIFORNIA EDISON®
An EDISON INTERNATIONAL® Company

Multiple Choice Questions

To answer each multiple choice question, you should move the mouse pointer over the circle (radio button) next to the answer of your choice, and click the left mouse button.

A sample is shown below:

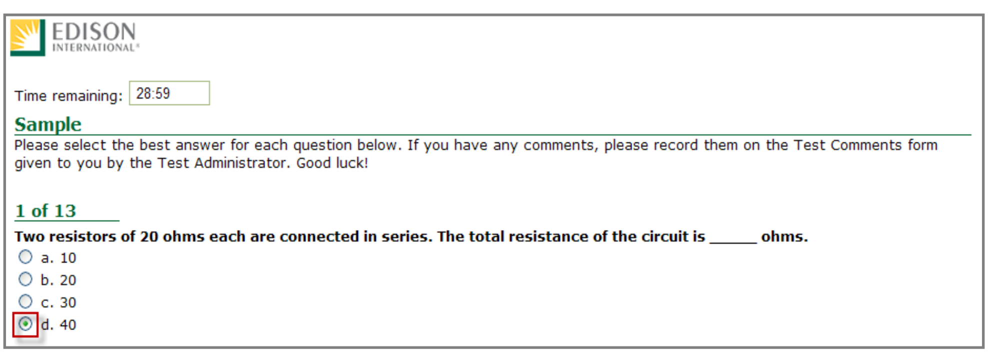
1. In order to answer each question, first read the question and determine the response that best answers the question. Put the mouse pointer directly over the circle corresponding to that response.



2. While the pointer is over the circle corresponding to the best answer, click the left mouse button.

Click the left button when the pointer icon is over your answer choice.

3. The answer you selected should now have a green dot in the circle. If you need to select an alternate answer, simply move the pointer over that circle, and click again.



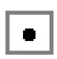
SOUTHERN CALIFORNIA EDISON®
An EDISON INTERNATIONAL® Company

Hot Spot Questions

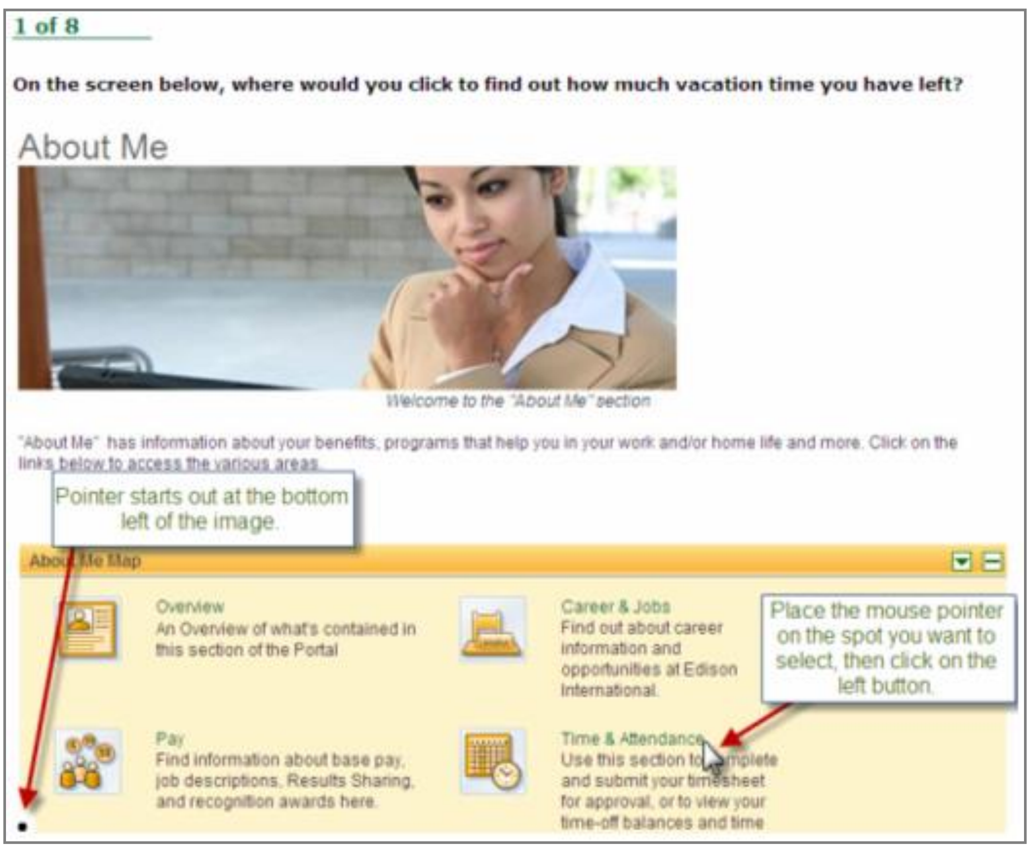
To answer each Hot Spot question, you should move the mouse pointer over the part of the image that best answers the question, and click the left mouse button. You will see a pointer appear in that spot. If you want to change your answer, simply move the mouse pointer to a new area on the picture and click again. The pointer will move to the new spot.

A sample is shown below:

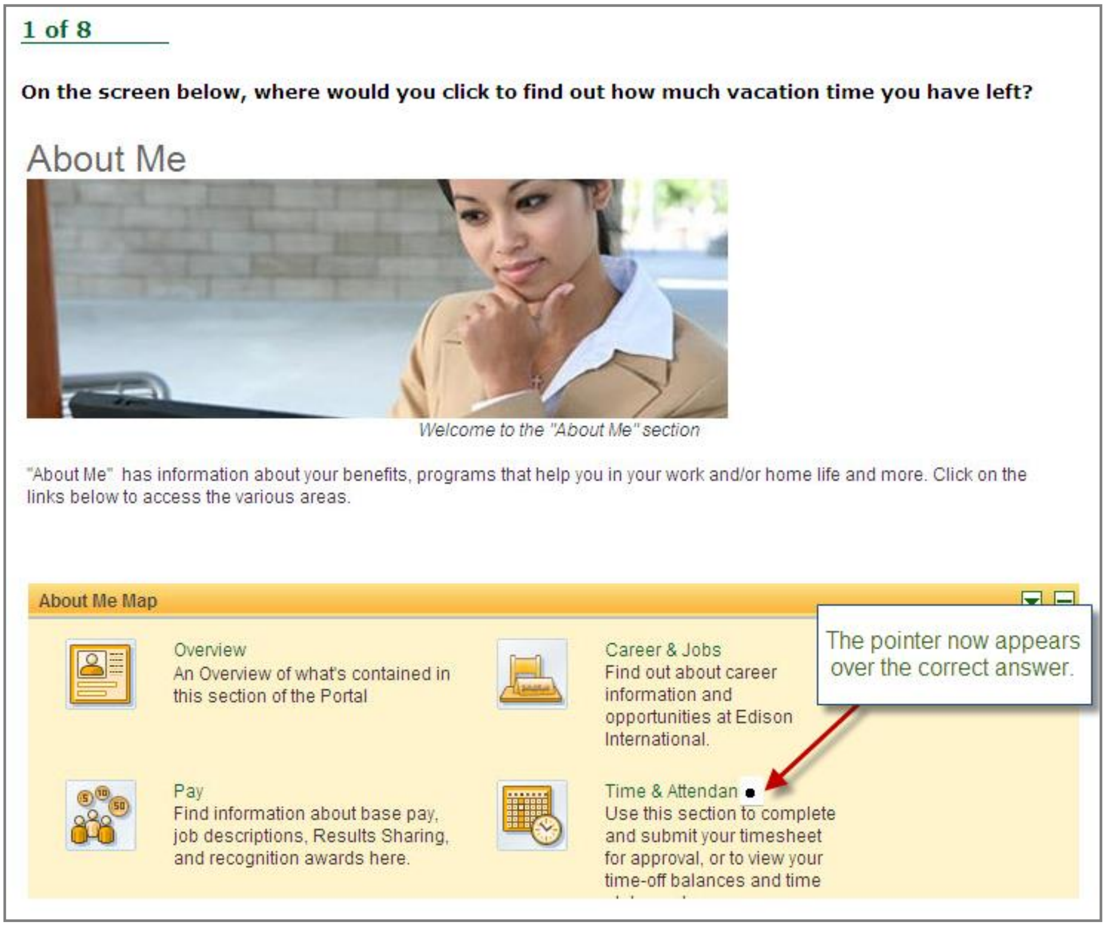
1. In order to answer each question, first read the question and determine the place on the image that best answers the question. The pointer that will indicate your answer can always be seen in the bottom left of the image. It looks like this:



Put the mouse pointer directly over the spot on the image you want to select, and click the left mouse button.



2. The pointer will move from the bottom left of the image and appear over the spot you selected.



3. To change your answer, simply move the mouse pointer to the new spot, and click again. The pointer graphic will move to the new spot you've selected. In order for your answer to be considered be correct, the center of the pointer (•) must be over the correct spot on the graphic.

Test Taking Strategies

Introduction

The test contains multiple-choice questions. The purpose of this section is to help you to identify some special features of a multiple-choice test and to suggest techniques for you to use when taking one.

Your emotional and physical state during the test may determine whether you are prepared to do your best. The following list provides common sense techniques you can use before the test begins.

Technique	Remarks
Be confident	- If you feel confident about passing the test, you may lose some of your anxiety.
	- Think of the test as a way of demonstrating how much you know, the skills you can apply, the problems you can solve, and your good judgment capabilities.
Be punctual	- Arrive early enough to feel relaxed and comfortable before the test begins.
Concentrate	- Try to block out all distractions and concentrate only on the test. You will not only finish faster but you will reduce your chances of making careless mistakes.
	- If possible, select a seat away from others who might be distracting.
	- If lighting in the room is poor, sit under a light fixture.
	- If the test room becomes noisy or there are other distractions or irregularities, mention them to the Test Administrator immediately.

Budget your times	-	Pace yourself carefully to ensure that you will have enough time to complete all items and review your answers.
Read critically	-	Read all directions and questions carefully.
	-	Even though the first or second answer choice looks good, be sure to read all the choices before selecting your answer.
Make educated guesses	-	Make an educated guess if you do not know the answer or if you are unsure of it.
Changing answers	-	If you need to change an answer, be sure to erase your previous answer completely. On the computer, be sure that the new answer is selected instead of the old one.
Return to difficult questions	-	If particular questions seem difficult to understand, make a note of them, continue with the test and return to them later.
Double-check math calculations	-	Use scratch paper to double check your mathematical calculations.
Review	-	If time permits, review your answers.
	-	Do the questions you skipped previously.
	-	Make sure each answer bubble is completely filled in. Erase any stray marks on your answer sheet. When testing on the computer, make sure each multiple choice question has a green dot next to the correct answer.

Remember the techniques described in this section are only suggestions. You should follow the test taking methods that work best for you.

Job Knowledge Categories and Study References

Below are the major job knowledge areas (topics) covered on the **2774 Test Technician Test**. Listed next to each knowledge category is the number of items on the exam that will measure that topic. You can use this information to guide your studying. Some exams also contain additional pretest items. Pretest items will appear just like all of the other items on your exam, but they will not affect your score. They are an essential part of ensuring the **2774 Test Technician Test** remains relevant to successful performance of the job.

There are a total of 101 items on the test and the passing score is 75%.

A. Electrical Theory (75 items)
Includes basic AC/DC theory, Ohm's law, Watt's law, Kirchhoff's Law, circuit diagrams, electrical symbols, and calculations for electrical circuits.

B. Electronic Theory (15 items)
Includes electronic components and electronic symbols.

C. Mathematics (8 items)
Trigonometry – knowledge of sine, cosine, tangent ratios, and their application in electrical theory (e.g., phasor angle); this includes the ability to solve triangle problems using trigonometric functions.

D. Test Instruments and Procedures (3 items)
Includes electrical measurement, AC meters, and basic instrument use.

Study References

1. https://www.allaboutcircuits.com/textbook/
2. https://www.allaboutcircuits.com/worksheets/
3. https://www.allaboutcircuits.com/video-lectures/
4. https://www.allaboutcircuits.com/technical-articles/
5. The Test Technician Study Guide Workbook (published by SCE Power Production Training) can be found as an appendix to this document.

Important Note: The knowledge categories described below are different from the knowledge categories that appear in the Test Technician Study Guide Workbook provided through SCE PPT. Please refer to the knowledge categories in this guide, not the knowledge categories in the workbook, for relevance to the test.

Study Guide Feedback

Please use this page to notify us of any changes in policies, procedures, or materials affecting this
guide. Once completed, return to:

Southern California Edison
Human Resources – Testing
G.O. 5, 1st Floor
1515 Walnut Ave.
Rosemead, CA 91770

Test Name: **2774 Test Technician Test**

Page	Comments
______	____
	____
	____
______	____
	____
	____
______	____
	____
	____
______	____
	____
	____

Appendix

Test Technician Study Guide Workbook

N O T I C E

These materials are protected by copyrights.

It is illegal to reproduce these training materials

or any part of them in any way, to share these materials

or to loan or rent them to anyone else without permission.

7301 Fenwick Lane Westminster, CA 92683

Table of Contents

Study Guide Outline
Job Knowledge Categories

Below are the major job knowledge categories that are covered on the test.

A. Electrical Theory

Includes AC and DC theory, Ohm's law, wiring and circuit diagrams, electrical symbols, 3 phase power theory and electrical terminology

B. Electronic Theory

Includes basic electronic theory, circuitry, electronic symbols, solid state theory, and knowledge of diodes, rectifiers, transistors, resonance, and logic symbols

C. Mathematics

Includes algebra, geometry, trigonometry, and phasoring

D. Test Instruments and Procedures

Refer to standard test procedures and accuracy requirements and the use of electrical test instruments, meters, and tools.

E. Equipment Knowledge

Refers to knowledge of electrical equipment including protective relays, meters, recording instruments, supervisory control equipment, transformers, voltage regulators, synchronous condensers, power circuit breakers, carrier current equipment, and other electrical equipment tested by the Test Technician

F. Safety

Includes knowledge of safety procedures, electrical hazards, first aid, fire fighting, and safe operating procedures, including clearance procedures

Study References

Below is a combined listing of the study references for material covered on the test. The materials listed in this Guide are available from public/university libraries, general bookstores, university or technical bookstores. Department reference material (e.g., operating letters, on-line computer systems, etc.) again will depend on project.

KNOWLEDGE CATEGORY A — ELECTRICAL THEORY

Basic Electricity
Bureau of Navy Personnel, Dover Publications

Delmar's Standard Textbook of Electricity
Delmar Cengage Learning, by Stephen L. Herman

Vector Analysis
Industrial Press, Stroud and Booth

KNOWLEDGE CATEGORY B — ELECTRONIC THEORY

Basic Solid State Electronics
Van Valkenburgh, Nooger & Neville. Inc
Substation Training School

Basic Electronics
Bernard Grob, McGraw Hill Book Co.

Electronic Principles

Albert Paul Malvino, Glencoe Macmillan/McGraw-Hill

KNOWLEDGE CATEGORY C — MATHEMATICS

Basic Mathematics For Electronics
Nelson M. Cooke, McGraw Hill Book Co.

Working with Numbers: Refresher Algebra
Janies T. Shea, STECK-VAUGN

Geometry: A Straightforward Approach
Martin M. Zuckerman, Morton Publishing Co.

Trigonometry the Easy Way
Douglas Downing, Barron's Education Service

KNOWLEDGE CATEGORY D — TEST INSTRUMENTS AND PROCEDURES

Basic Electricity
Bureau of Navy Personnel,
Substation Training School

Delmar's Standard Textbook of Electricity
Delmar Cengage Learning, by Stephen L. Herman

Math
(General Physics)

Trigonometry

Trigonometry is the study of angles and the relationship between angles and the lines that form them. *All trigonometry is based on a right-angled triangle.* The most important application of trigonometry is the solution of triangles based on the sizes of the angles and the lengths of the sides. This lesson explains:

- Sines
- Cosines
- Tangents

Objectives

After successfully completing this lesson, you will be able to:

1. *Define* the sine, cosine, and tangent ratios.
2. *Graph* the sine and cosine functions.
3. *Solve* triangle problems using trigonometric functions.

Key Words

Sin – Abbreviation of sine.

Cos – Abbreviation of cosine.

Tan – Abbreviation of tangent.

The lengths of the sides of a right triangle are related to the size of the angle θ. This is the basis for trigonometric (trig) functions.

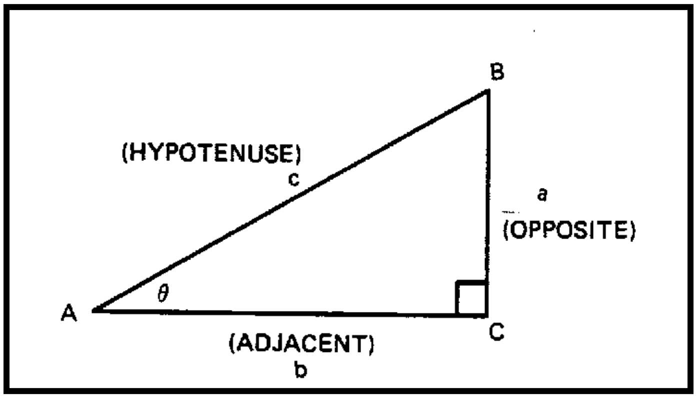


Figure 28. Right Triangle for Trig Functions

The sides of the triangle in Figure 28 are labeled based on the angle about which you are talking, A or θ (theta) in this case. The side opposite θ is labeled "a." The side next to or adjacent to θ is labeled "b." The hypotenuse is still called the hypotenuse and is labeled "c." The angle opposite the hypotenuse, or the right angle (90°), is labeled "C."

The ratio of the opposite side to the hypotenuse is called the sine of angle θ. Sine is abbreviated **sin**.

$$\sin \theta = \frac{\textit{length of opposite side}}{\textit{length of hypotenuse}}$$

or

$$\sin \theta = \frac{a}{c}$$

The reciprocal of sin θ, the length of the hypotenuse divided by the length of the opposite side, is called the cosecant, or CSC θ.

If you know any two parts of the sine equation, you can easily calculate the third. If you know the sine of an angle and want to find the angle itself, you can write this as $\sin^{-1}$ or arcsin.

If you calculate the value of sin θ for various θs from 0° to 360°, you could plot them as in Figure 29. This is called a sine curve. Notice that sin θ is never greater than +1 or less than -1. The values of all trig functions for angles from 0° to 90° have been calculated and are listed in standard trig function tables. As with log tables, fractions of degrees can be interpolated.

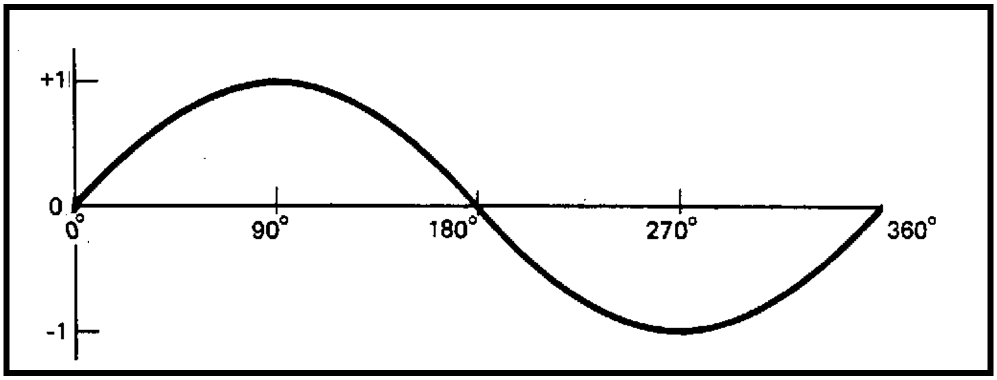
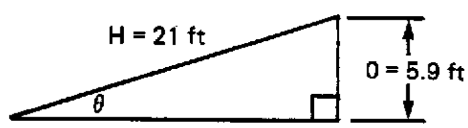


Figure 29. Sine Curve

Look at a sample problem: A plank 21 ft. long is used to roll a barrel onto a truck. If the truck bed is 5.9 ft. above the ground, what angle does the plank form with the ground?

Solution:



$$\sin\theta = \frac{5.9}{21}$$

$$\theta = \sin^{-1}\frac{5.9}{21}$$

$$= \sin^{-1} 0.2809$$

From standard trig tables, or using a calculator with trig functions, $\sin^{-1} 16.3° = 0.2807$, which is very close to 0.2809.

Therefore $\theta \approx 16.3°$

It is usually helpful to draw a simple diagram of problems.

The ratio of the adjacent side to the hypotenuse is called the cosine of θ. Cosine is abbreviated **cos.**

$$\cos\theta \;=\; \frac{\textit{length of adjacent side}}{\textit{length of hypotenuse}}$$

or

$$\cos\theta \;=\; \frac{b}{c}$$

The reciprocal of the cos θ, the length of the hypotenuse divided by the length of the adjacent side, is called the secant, or sec θ. See Figure 28, repeated below.

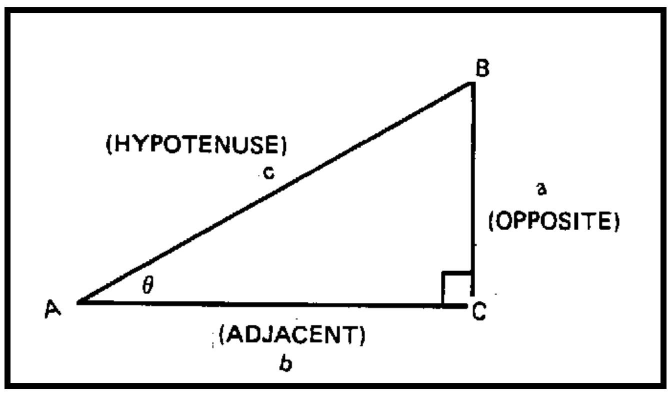


Figure 28. Right Triangle for Trig Functions

The angle whose cosine is known is written $\cos^{-1}$ or arccos.

A plot of cos θ for θ from 0° to 360° looks like Figure 30. Notice that cos θ is never larger than +1 or less than -1.

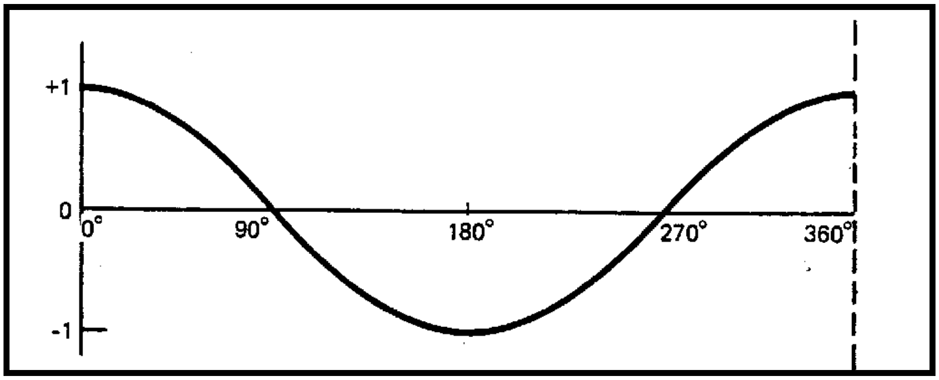
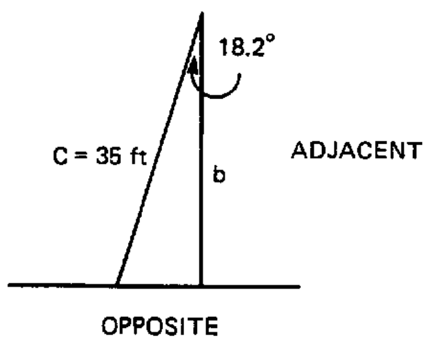


Figure 30. Cosine Curve

Look at a sample problem: How high will a 35 ft. long ladder reach up a vertical wall if it makes an angle of 18.2° with the wall?

Solution:



$$\cos 18.2° = \frac{b}{35}$$

$$b = 35 \cos 18.2°$$

From standard trig tables, or using a calculator with trig functions, cos 18.2° = 0.9500.

$$b = 35\ (0.95)$$

$$= 33.25 \text{ ft.}$$

Tangent

The ratio of the opposite side to the adjacent side is called the tangent of θ. Tangent is abbreviated **tan.**

$$\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

or

$$\cos\theta = \frac{O}{A}$$

The reciprocal of tan θ, the length of the adjacent side divided by the length of the opposite side, is called the cotangent, or cot θ. See Figure 28, repeated below.

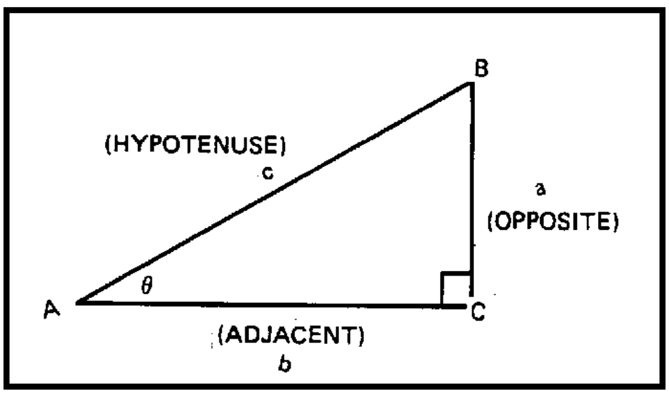
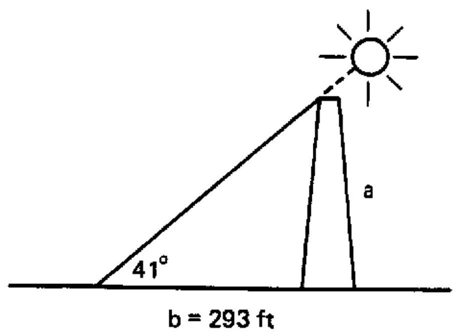


Figure 28. Right Triangle for Trig Functions

The angle whose tangent is known is written $\tan^{-1}$ or arctan. The tangents of 90° and 270° are undefined.

Look at a sample problem: The shadow of a stack is 293 ft. long when the sun is at 41° elevation. Find the height of the stack.

Solution:



$$\tan 41° = \frac{a}{293}$$

$$a = 293 \tan 41°$$

From standard trig tables, or using a calculator with trig functions, tan 41° = 0.8693.

$$a = 293\,(0.8693)$$

$$= 254.7 \text{ ft.}$$

Laws of Sines and Cosines

The solutions of all triangles can be grouped into cases according to the information given about the angles and sides.

Section 1. Law of Sines

The Law of Sines states that in any triangle ABC, the sides are proportional to the sines of the opposite angles. See Figure 31 for an example of a typical triangle ABC.

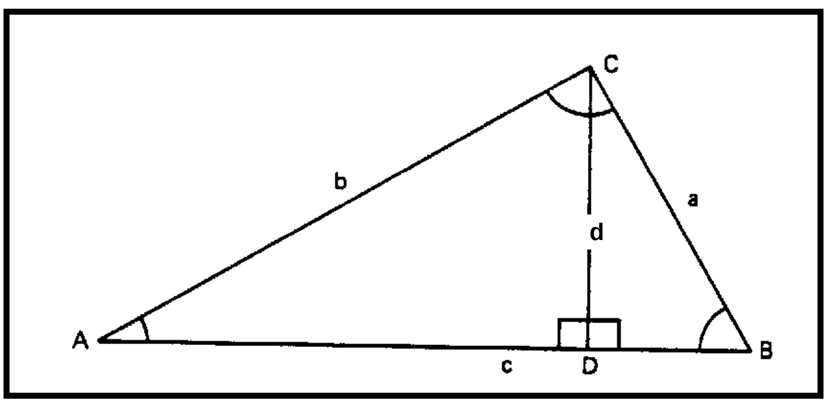


Figure 31. Typical Triangle ABC

$$\sin A = \frac{d}{b} \qquad \sin B = \frac{d}{a}$$

therefore $d = b \sin A$ $\qquad d = a \sin B$

Since $d = d$, it is obvious then that to solve for b:

$$b \sin A = a \sin B$$

Dividing by sin A

$$\frac{b \sin A}{\sin A} = \frac{a \sin B}{\sin A}$$

$$b = \frac{a \sin B}{\sin A}$$

Dividing by sin B

$$\frac{b}{\sin B} = \frac{a \sin B}{\sin A \sin B}$$

or

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

The Law of Sines is normally written as shown below:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Derivations can be obtained from the above relationships as follows:

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{c}{-\sin C} = \frac{a}{\sin A}$

a sin B = b sin A b sin C = c sin B a sin C = c sin A

sin A = $\frac{a \sin B}{b}$ sin B = $\frac{b \sin C}{c}$ sin C = $\frac{c \sin A}{a}$

The Law of Cosines states that in any triangle ABC, the square of any side is equal to the sum of the squares of the other two sides diminished by twice the product of the other two sides and the cosine of the included angle. See Figure 31, repeated below.

Using the Pythagorean Theorem in the left triangle:

$$b^2 = d^2 + (AD)^2$$

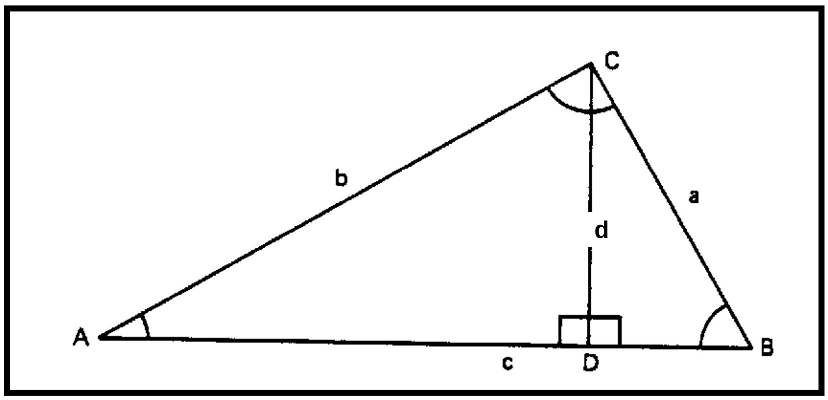


Figure 31. Typical Triangle ABC

In the triangle on the right side:

$$\sin B = \frac{d}{a} \qquad \cos B = \frac{DB}{a}$$

then $d = a \sin B$ $\qquad DB = a \cos B$

Then $AD = AB - DB = c - a\cos B$

and $b^2 = d^2 + (AD)2 = (a \sin B)^2 + (c - a \cos B)^2$

or $b^2 = a^2 \sin^2 B + (c^2 - 2ac \cos B + a^2 \cos^2 B)$

$b^2 = a^2 \sin^2 B + c^2 - 2ac \cos B + a^2 \cos^2 B$

$b^2 = a^2 (\sin^2 B + \cos^2 B) + c^2 - 2ac \cos B$

It can be shown from the Pythagorean Theorem that:

$$(\sin^2 B + \cos^2 B) = 1$$

Therefore: $b^2 = a^2 + c^2 - 2ac \cos B$

The relationships for finding other values can be derived in the above manner to yield:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

To determine unknown angles, the above relationships can be manipulated to yield the following equations:

To find angle C:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$0 = a^2 + b^2 - c^2 - 2ab \cos C$$

$$2ab \cos C = a^2 + b^2 - c^2$$

$$\frac{2\ ab\ \cos\ C}{2\ ab} = \frac{a^2 + b^2 - c^2}{2\ ab}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2\ ab}$$

Similarly,

$$\cos A = \frac{b^2 + c^2 - a^2}{2\ bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2\ ac}$$

NOTE: In general, the Law of Sines is used when two angles and one side or two sides and an angle opposite one of them are known, while the Law of Cosines is used when two sides and the included angle or three sides are known.

Practice problems using the trig functions are in the next section.

Use standard trig table functions or, if you have one, a calculator with trig functions, to solve these problems. Answers are at the end of the module.

1. A road makes an angle of 7.4° with the horizontal. Find the increase in elevation (in feet) if you drive one mile. (One mile = 5280 ft.)

2. A 30 ft. ladder is placed against a vertical wall so that the foot of the ladder is 6.5 ft. from the wall. What angle does the ladder make with the ground? How high on the wall does the ladder reach?

3. Given the following, find the angles:

   a. arcsin 0.1564

   b. arccos 0.301

   c. arctan 0.419

   d. $\sin^{-1}$ 0.0262

   e. $\tan^{-1}$ 0.0115

   f. $\tan^{-1}$ 2.05

4. A platform is 10 ft. above floor level. A ramp is to be built from the floor to the platform. If the ramp is to make an angle with the floor of 14°, how far from the platform must the ramp start? How long must the ramp be?

1. A tower is braced by a cable fastened 15 ft. below the top and to an anchor that is 65 ft. from the base of the tower. If the brace makes an angle of 70° with the ground, how high is the tower?

Electronics

Transformer and Diode Rectifier Circuits

A rectifier diode is ideally a closed switch when forward-biased and an open switch when reverse-biased. Because of this, it is useful for converting alternating current to direct current. This chapter discusses three basic rectifier circuits called the half-wave rectifier, the full-wave rectifier, and the bridge rectifier.

The Input Transformer

Power companies in the United States supply a nominal line voltage of 115 V rms at a frequency of 60 Hz. The actual voltage coming out of a power outlet may vary from 105 V to 125 V rms, depending on the time of day, locality, and other factors. Recall that the relation between the rms value and the peak value of a sine wave is given by

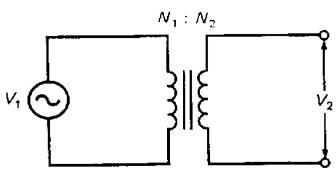
$$V_{rms} = 0.707V_p \qquad (4\text{-}1)$$

This equation says that the rms voltage equals 70.7 percent of the peak voltage. Recall what rms value means. This is the equivalent dc voltage that would produce the same amount of power over one complete cycle.

Basic Equation

Line voltage is too high for most of the devices used in electronics equipment. This is why a transformer is commonly used in almost all electronics equipment. This transformer steps the ac voltage down to lower levels that are more suitable for use with devices like diodes and transistors.

Figure 4-1 shows an example of a transformer. The left coil is called the *primary winding* and the right coil is called the *secondary winding*. The number of turns on the primary winding is N_1, and the number of turns on the secondary winding is N_2. The vertical lines between the primary and secondary windings indicate that the turns are wrapped on an iron core.



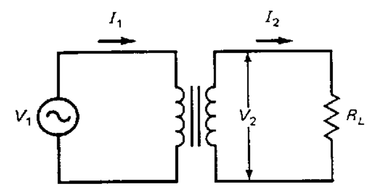


Figure 4-1
Unloaded Transformer

Figure 4-2
Loaded Transformer

With this type of transformer, the coefficient of coupling *k* approaches one, which means tight coupling exists. In other words, all the flux produced by the primary winding cuts through the secondary winding. The voltage induced in the secondary winding is given by

$$V_2 = \frac{N_2}{N_1} V_1 \qquad (4\text{-}2)$$

The voltages in this equation may be either rms or peak voltages. Just be consistent and use rms for both, or peak for both.

Step-Up Transformer

When the secondary winding has more turns than the primary winding, more voltage is induced in the secondary than in the primary. In other words, when N_2/N_1 is greater than one, the transformer is referred to as a step-up transformer. If N_1 = 100 turns and N_2 = 300 turns, the same flux cuts through three times as many turns in the secondary as in the primary winding. This is why the secondary voltage is three times as large as the primary voltage.

Step-Down Transformer

When the secondary winding has fewer turns than the primary winding, less voltage is induced in the secondary than in the primary. In this case, the turns ratio, N_2:N_1, is less than one, and the transformer is called a step-down transformer. If N_1 = 100 turns and N_2 = 50 turns, the same flux cuts through half as many turns in the secondary as in the primary winding, and the secondary voltage is half the primary voltage.

Effect on Current

Figure 4-2 shows a load resistor connected across the secondary winding. Because of the induced voltage across the secondary winding, a current exists. If the transformer is ideal (k = 1 and no power is lost in the windings or the core), the output power equals the input power:

$$P_2 = P_1$$

or

$$V_2 I_2 = V_1 I_1$$

We can rearrange the foregoing equation as follows:

$$\frac{I_1}{I_2} = \frac{V_2}{V_2}$$

But Eq. (4-2) implies that $V_2/V_1 = N_2/N_1$. Therefore,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

or

$$I_1 = \frac{N_2}{N_1} I_2 \qquad (4\text{-}3)$$

An alternative way to write the foregoing equation is

$$I_2 = \frac{N_1}{N_2} I_1 \qquad (4\text{-}4)$$

Notice the following. For a step-up transformer, the voltage is stepped up but the current is stepped down. On the other hand, for a step-down transformer, the voltage is stepped down but the current is stepped up.

Example 4-1

Suppose the voltage from a power outlet is 120 V rms. What is the peak voltage?

Solution

Using algebra, we can rewrite Eq. (4-1) in this equivalent form:

$$Vp = \frac{V_{rms}}{0.707}$$

Now, substitute the rms voltage and calculate the peak voltage:

$$Vp = \frac{120\text{V}}{0.707} = 170\text{V}$$

This tells us that the sinusoidal voltage out of the power outlet has a peak value of 170 V.

Example 4-2

A step-down transformer has a turns ratio of 5:1. If the primary voltage is 120 V rms, what is the secondary voltage?

Solution

Divide the primary voltage by 5 to get the secondary voltage:

$$V_2 = \frac{120\text{V}}{5} = 24\text{V}$$

Example 4-3

Suppose a step-down transformer has a turns ratio of 5:1. If the secondary current is 1 A rms, what is the primary current?

Solution

With Eq. (4-3),

$$I_1 = \frac{1\text{A}}{5} = 0.2\text{A}$$

As a check on this answer, use your common sense as follows, This is a step-down transformer, which means the current is stepped up going from primary to secondary, equivalent to saying the current is stepped down as we go from the secondary to the primary. This means the primary current is five times smaller than the secondary current. Whenever possible, you should check that your answers are logical because it is easy to make a mistake with equations.

The Half-Wave Rectifier

The simplest circuit that can convert alternating current to direct current is the half-wave rectifier, shown in Fig. 4-3. Line voltage from an ac power outlet is applied to the primary winding of the transformer. Usually, the power plug has a third prong to ground the equipment. Because of the turns ratio, the peak voltage across the secondary winding is

$$V_{p2} = \frac{N_2}{N_1} V_{p1}$$

Recall the dot convention used with transformers. The dotted ends of a transformer have the same polarity of voltage at any instant in time. When the upper end of the primary winding is positive, the upper end of the secondary winding is also positive. When the upper end of the primary winding is negative, the upper end of the secondary winding is also negative.

Here is how the circuit works. On the positive half cycle of primary voltage, the secondary winding has a positive half sine wave across it. This means the diode is forward-biased. However, on the negative half cycle of primary voltage, the secondary winding has a negative half sine wave. Therefore, the diode is reverse-biased. If you use the ideal-diode approximation for an initial analysis, you will realize that the positive half cycle appears across the load resistor, but not the negative half cycle.

For instance, Fig. 4-4 shows a transformer with a turns ratio of 5:1. The peak primary voltage is

$$V_{p1} = \frac{120\text{V}}{0.707} = 170\text{V}$$

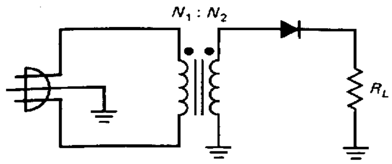


Figure 4-3
Half-wave Rectifier

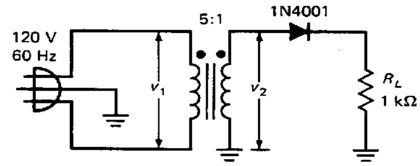


Figure 4-4
5:1 Turns Ratio

The peak secondary voltage is

$$V_{p2} = \frac{170\text{V}}{5} = 34\text{V}$$

With the ideal-diode approximation, the load voltage has a peak value of 34 V.

Figure 4-5 shows the load voltage. This type of waveform is called half-wave signal because the negative half cycles have been clipped off or removed. Since the load voltage has only a positive half cycle, the load current is unidirectional, meaning that it flows only in one direction. Therefore, the load current is a pulsating direct current. It starts at zero at the beginning of the cycle, then increases to a maximum value at the positive peak, then decreases to zero where it sits for the entire negative half cycle.

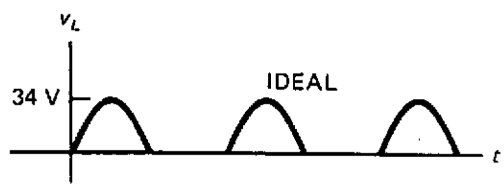


Figure 4-5
Half-wave Signal

Period

The frequency of the half-wave signal is still equal to the line frequency, which is 60 Hz. (In Europe, line frequency is 50 Hz.) Recall that the period, T, equals the reciprocal of the frequency. Therefore, the half-wave signal has a period of

$$T = \frac{1}{f} = \frac{1}{60\text{ Hz}} = 0.0167\text{s} = 16.7\text{ms}$$

This is the amount of time between the beginning of a positive half cycle and the start of the next positive half cycle. This is what your would measure if you looked at a half-wave signal with an oscilloscope.

DC or Average Value

If you connect a dc voltmeter across the load resistor of Figure 4-5, it will indicate a dc voltage of $V_{p/\pi}$, which may be written as

$$V_{dc} = 0.318V_p \qquad (4\text{-}5)$$

where V_p is the peak value of the half-wave signal across the load resistor. For instance, if the peak voltage is 34 V, the dc voltmeter will read

$$V_{dc} = 0.318(34\ \text{V}) = 10.8\ \text{V}$$

This dc voltage is sometimes called the "average" value of the half-wave signal because the voltmeter reads the average voltage over one complete cycle. The needle of the voltmeter cannot follow the rapid variations of the half-wave signal, so the needle settles down on the average value, which is 31.8 percent of the peak value. (The 31.8 percent can be proved with calculus.)

Approximations

Because the secondary voltage is much greater than the knee voltage, using the second approximation will improve the analysis only slightly. If we use the second approximation, the half-wave signal has a peak of 33.3 V. Furthermore, since the bulk resistance of a 1N4001 is only 0.23 Ω compared to a load resistance of 1 kΩ, there is no increase in accuracy when using the third approximation. In conclusion, either the ideal diode or the second approximation is adequate in analyzing this circuit.

Example 4-4

In Europe, a half-wave rectifier has an input voltage of 240 V rms with a frequency of 50 Hz. If the step-down transformer has a turns ratio of 8:1, what is the load voltage?

Solution

You can divide 240 V by 0.707 to get the answer. Here is an alternative way to get the peak voltage. Since the rms voltage is twice as large as previous examples, the peak voltage is twice as large as before:

$$V_{pl} = 2(170\text{ V}) = 340\text{ V}$$

Because of the 8:1 step down, the secondary voltage has a peak value of

$$V_{p2} = \frac{340\text{V}}{8} = 42.5\text{V}$$

Ignoring the diode drop means that the load voltage is a half-wave signal with a peak value of 42.5 V.

The period of the rectified output voltage is slightly longer:

$$T = \frac{1}{50\text{Hz}} = 0.02\text{s} = 20\text{ms}$$

This is what you would measure with an oscilloscope.

The Full-Wave Rectifier

Figure 4-6 shows a "full-wave rectifier." Notice the grounded center tap on the secondary winding. Because of this center tap, the circuit is equivalent to two half-wave rectifiers. The upper rectifier handles the positive half cycle of secondary voltage, while the lower rectifier handles the negative half cycle of secondary voltage. In other words, D_1 conducts on the positive half cycle and D_2 conducts on the negative half cycle. Because of this, the rectified load current flows during both half cycles. Furthermore, this load current flows in one direction only.

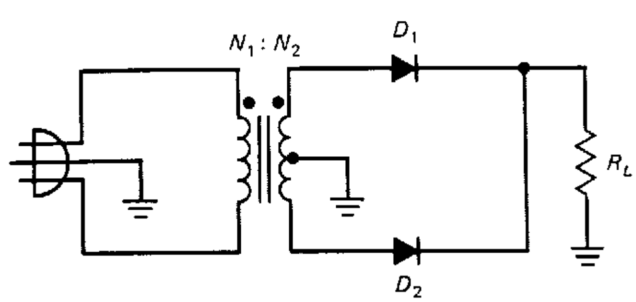


Figure 4–6
Full-wave Rectifier

For instance, Fig. 4-7 shows a transformer with a turns ratio of 5:1. The peak primary voltage is still equal to

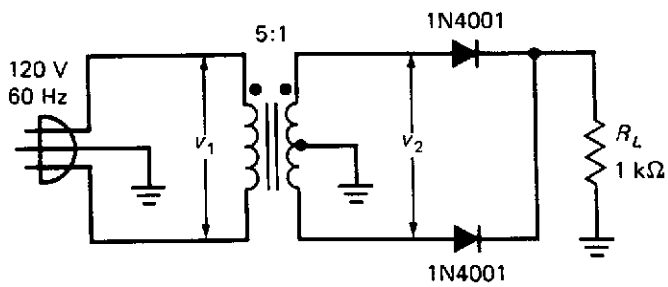


Figure 4–7
Example of Full-wave Rectifier

$$V_{p1} = \frac{120\text{V}}{0.707} = 170\text{V}$$

The peak secondary voltage is

$$V_{p2} = \frac{170\text{V}}{5} = 34\text{V}$$

Because of the grounded center tap, each half of the secondary winding has a sinusoidal voltage with a peak of only 17 V. Therefore, the load voltage has an ideal peak value of only 17 V instead of 34 V. This factor- of-two reduction is a characteristic of all full-wave rectifiers. It is a direct result of using a grounded center tap on the secondary winding.

Figure 4-8 shows the load voltage. This type of waveform is called a full-wave signal. It is equivalent to inverting or flipping the negative half cycles of a sine wave to get positive half cycles. Because of Ohm's law, the load current is a full-wave signal with a peak value of

$$I_p = \frac{17\text{V}}{1\text{k}\Omega} = 17\text{mA}$$

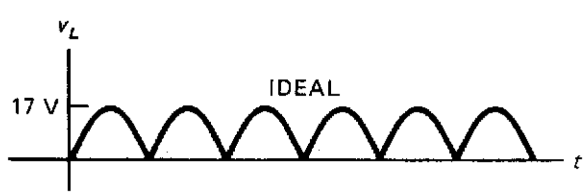


Figure 4–8
Full-wave Signal

DC or Average Value

If you connect a dc voltmeter across the load resistor of Fig. 4-7, it will indicate a dc voltage of $2V_{p/\pi}$, which is equivalent to

$$V_{dc} = 0.636V_p \qquad (4\text{-}6)$$

where V_p is the peak value of the half-wave signal across the load resistor. For instance, if the peak voltage is 17 V, the dc voltmeter will read

$$V_{dc} = 0.636(17\text{ V}) = 10.8\text{ V}$$

This dc voltage is the average value of the full-wave signal because the voltmeter reads the average voltage over one complete cycle.

Output Frequency

The frequency of the full-wave signal is double the input frequency. Why? Recall how a complete cycle is defined. A waveform has a complete cycle when it repeats. In Fig. 4-8, the rectified waveform begins repeating after one half cycle of the primary voltage. Since line voltage has a period, T_1, of

$$T_1 = \frac{1}{f} = \frac{1}{60\text{Hz}} = 0.0167\text{s} = 16.7\text{ms}$$

The rectified load voltage has a period, T_2, of

$$T_2 = \frac{16.7\text{ms}}{2} = 8.33\text{ms}$$

The frequency of the load voltage therefore equals

$$f_2 = \frac{1}{T_2} = \frac{1}{8.33\text{ms}} = 120\text{Hz}$$

This says the output frequency equals two times the input frequency. In symbols,

$$f_{out} = 2f_{in} \qquad (4\text{-}7)$$

This doubling of the frequency is a characteristic of all full-wave rectifiers. It is a direct result of using two diodes, one to rectify the positive half cycle of input voltage and the other to rectify the negative half cycle of input voltage. Visually, the effect is to invert the negative half of the input voltage to get a full-wave signal.

Again, notice the following about the use of diode approximations. Because the secondary voltage is much greater than the knee voltage, the second approximation results in a full-wave output voltage with a peak value of 16.3 V instead of 17 V, Once more, the small bulk resistance of a 1N4001 has almost no effect. In conclusion, either the ideal diode or the second approximation is adequate in analyzing most full-wave circuits. The only time you would consider using the third approximation is when the load resistance is small.

Example 4-5

Suppose the full-wave rectifier of Fig, 4-7 has an input voltage of 240 V rms with a frequency of 50 Hz. If the step-down transformer has a turns ratio of 8:1, what is the load voltage?

Solution

The peak primary voltage is the same as the previous example::

$$V_{p1} = 340\text{V}$$

The peak secondary voltage has the same peak value as before:

$$V_{p2} = 42.5\text{V}$$

The center tap reduces this voltage by a factor of 2. In other words, the entire secondary winding has a sine wave across it with a peak value of 42.5 V. Therefore, each half of the secondary winding has a sine wave with only half this peak value, or approximately 21.2 V. Ignoring the diode drop means that the load voltage is a full-wave signal with a peak value of 21.2 V.

Also, the rectified output signal has a frequency of twice the input frequency. In this case, the output frequency is

$$f = 2(50\text{ Hz}) = 100\text{ Hz}$$

Figure 4-9 shows a *bridge rectifier*. By using four diodes instead of two, this clever design eliminates the need for a grounded center tap. The advantage of not using a center tap is that the rectified load voltage is twice what it would be with the full-wave rectifier.

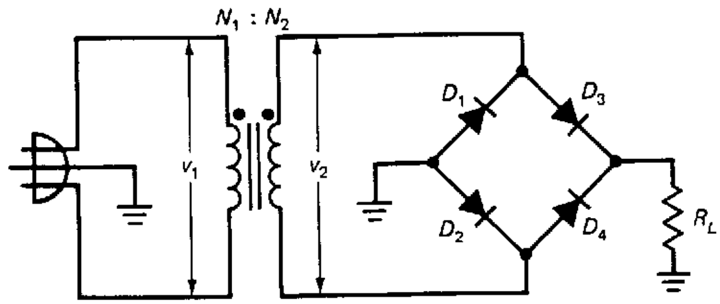


Figure 4–9
Bridge Rectifier

Here is how it works. During the positive half cycle of line voltage, diodes D_2 and D_3 conduct; this produces a positive half cycle across the load resistor. During the negative half cycle of line voltage, diode D_1 and D_4 conduct; this produces another positive half cycle across the load resistor. The result is a full-wave signal across the load resistor.

For instance, Fig. 4-10 shows a transformer with a turns ratio of 5:1. The peak primary voltage is still equal to

$$V_{p1} = \frac{120\text{V}}{0.707} = 170\text{V}$$

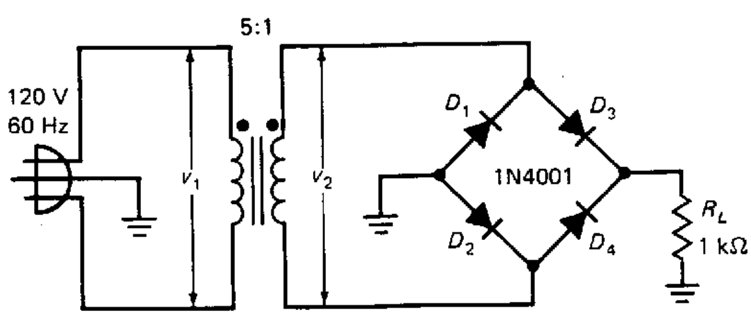


Figure 4–10
Example of Bridge Rectifier

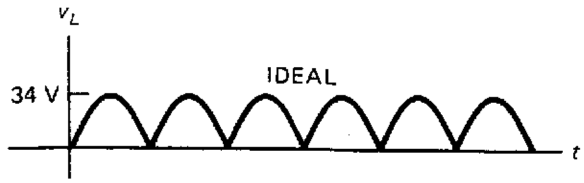


Figure 4-11
Full-wave Signal

The peak secondary voltage is still

$$V_{p2} = \frac{170\text{V}}{5} = 34\text{V}$$

Because the full secondary voltage is applied to the conducting diodes in series with the load resistor, the load voltage has an ideal peak value of 34 V, twice that of the full-wave rectifier discussed earlier.

Figure 4-11 shows the ideal load voltage. As you see, the shape is identical to that of a full-wave rectifier. Therefore, the frequency of the rectified signal equals 120 Hz, twice the line frequency. Because of Ohm's law, the load current is a full-wave signal with a peak value of

$$I_p = \frac{34\text{V}}{1k\Omega} = 34\text{mA}$$

There is a new factor to consider when using the second approximation with a bridge rectifier: there are two conducting diodes in series with the load resistor during each half cycle, Therefore, we must subtract two diode drops instead of only one, This means the peak voltage with the second approximation is

$$V_p, = 34\text{ V} - 2(0.7\text{ V}) = 32.6\text{ V}$$

The additional voltage drop across the second diode is one of the few disadvantages of the bridge rectifier, Also, there are two bulk resistances in series with the load resistance. But the effect is again negligible with the circuit values shown in Fig, 4-10. Unless you are designing a bridge rectifier, you will not normally use the third approximation because the bulk resistance is usually much smaller than the load resistance.

Most designers feel that having two diode drops and two bulk resistances is only a minor disadvantage. The advantages of the bridge rectifier include a full-wave output, an ideal peak voltage equal to the peak secondary voltage, and no center tap on the secondary winding. These advantages have made the bridge rectifier the most popular rectifier design. Most equipment uses a bridge rectifier to convert the ac line voltage to a dc voltage suitable for use with semiconductor devices.

Example 4-6

Suppose the bridge rectifier of Fig. 4-9 has an input voltage of 240 V rms with a frequency of 50 Hz. If the step-down transformer has a turns ratio of 8:1, what is the load voltage?

Solution

The peak primary voltage is the same as the previous example:

$$V_{p1} = 340\text{V}$$

The peak secondary voltage has the same peak value as before:

$$V_{p2} = 42.5\text{V}$$

This time, the entire secondary voltage is across two conducting diodes in series with the load resistor. Ignoring the diode drop means that the load voltage is a full-wave signal with a peak value of 42.5 V. Also, the frequency of the rectified output voltage is 100 Hz.

The Capacitor-Input Filter

The load voltage out of a rectifier is pulsating rather than steady. For instance, look at Fig. 4-11. Over one complete output cycle, the load voltage increases from zero to a peak, then decreases back to zero. This is not the kind of dc voltage needed for most electronic circuits. What is needed is a steady or constant voltage similar to what a battery produce. To get this type of rectified load voltage, we need to use a "filter."

Half-wave Filtering

The most common type of filter is the *capacitor-input* filter shown in Fig. 4-12. To simplify the initial discussion of filters, we have represented an ideal diode by a switch. As you can see, a capacitor has been inserted parallel with the load resistor. Before the power is turned on, the capacitor is uncharged; therefore, the load voltage is zero. During the first quarter cycle of the secondary voltage, the diode is forward-biased. Ideally, it looks like a closed switch. Since the diode connects the secondary winding directly across the capacitor, the capacitor charges to the peak voltage, V_p.

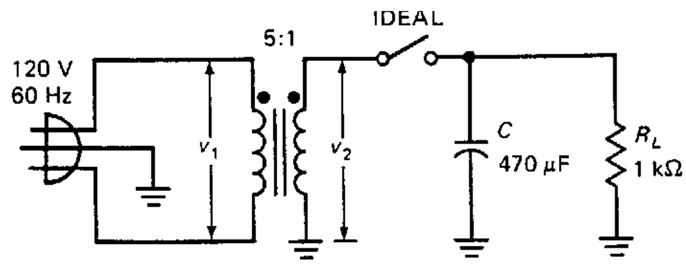


Figure 4–12
Capacitor-input Filter

Just past the positive peak, the diode stops conducting, which means the switch opens. Why? Because the capacitor has V_p volts across it. Since the secondary voltage is slightly less than V_p, the diode goes into reverse bias. With the diode now open, the capacitor discharges through the load resistance. But here is the key idea behind the capacitor-input filter: by deliberate design, the discharging time constant (the product of R_L and C) is much greater than the period, T, of the input signal. Because of this, the capacitor will lose only a small part of its charge during the off time of the diode as shown in Fig. 4-13a.

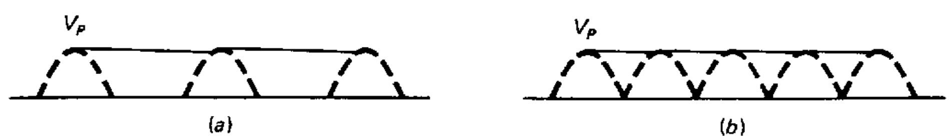


Figure 4–13
(a) Half-wave Filtering (b) Full-wave Filtering

When the source voltage again reaches its peak, the diode conducts briefly and recharges the capacitor to the peak voltage. In other words, after the capacitor is initially charged during the first quarter cycle, its voltage is approximately equal to the peak secondary voltage. This is why the circuit is sometimes called a *peak detector*.

The load voltage is now almost a steady or constant dc voltage. The only deviation from a pure dc voltage is the small ripple caused by charging and discharging the capacitor. The smaller the ripple is, the better. One way to reduce this ripple is by increasing the discharging time constant, which equals $R_L C$.

Full-Wave Filtering

Another way to reduce the ripple is to use a full-wave rectifier or bridge rectifier; then the ripple frequency is 120 Hz instead of 60 Hz. In this case, the capacitor is charged twice as often and has only half the discharge time (see Fig. 4-13b). As a result, the ripple is smaller and the dc output voltage more closely approaches the peak voltage. From now on, our discussion will emphasize the bridge rectifier driving a capacitor-input filter because this is the most commonly used circuit.

Brief Conduction of Diode

In the unfiltered rectifiers discussed earlier, each diode conducts for half a cycle. In the filtered rectifiers we are now discussing, each diode conducts for much less than half a cycle. When the power switch is first turned on, the capacitor is uncharged. Ideally, it takes only a quarter of a cycle to charge the capacitor to the peak secondary voltage. After this initial charging, the diodes turn on only briefly near the peak and are off during the rest of the cycle. In terms of degrees, the diodes turn on for only a couple of degrees during each cycle (half a cycle is $180°$).

An Important Formula

Whether you are troubleshooting, analyzing, or designing, you have got to know how to estimate the size of the ripple. Normally, the ripple is small compared to the peak secondary voltage. For most applications, the ripple is considered small when it is less than 10 percent of the load voltage. For instance, if the load voltage is I5 V, the ripple in most filtered rectifiers will be less than 1.5 V peak-to-peak.

Here is the formula for ripple expressed in terms of easily measured circuit values:

$$V_R = \frac{I}{fC} \qquad (4\text{-}8)$$

where V_R = peak-to-peak ripple voltage

I = dc load current

f = ripple frequency

C = capacitance

The proof of Eq. (4-8) is too lengthy and complicated to show in this book. But the derivation assumes that the peak-to-peak ripple is less than 20 percent of the load voltage. Beyond this point, you cannot use Eq. (4-8) without encountering a lot of error. But as was already discussed earlier, the whole point of the capacitor-input filter is to produce a steady or constant dc voltage. For this reason, most designers deliberately select circuit values to keep the ripple less than 10 percent of the load voltage. In the circuits you encounter, you will find that the ripple is usually less than 10 percent of the load voltage.

DC Voltage

To be successful in electronics, you have to learn the following basic idea: approximations are the rule, not the exception. Why? Because electronics is not an exact science like pure mathematics. The idea that you must always get exact answers is a false idea, a left-brain trap. For most of the work in electronics, approximate answers are adequate and even desirable.

The situation is like an artist painting a picture. The best artist starts with the largest brush when beginning a painting. The artist then switches to a medium-sized brush to improve the picture, and, finally, may use the smallest brush to get the finest detail. No good artist ever uses a small brush all of the time.

The three diode approximations are like an artist's brushes. You should start with the ideal diode to get the big picture. In many cases (trouble shooting, for instance), this will be all you need. Often, you will want to improve your analysis by using the second approximation (a lot of everyday work is done with this one). Finally, the third approximation may be best in some situations (if the circuit uses 1 percent resistors, for example).

First Approximation

With the foregoing in mind, here is how the diode approximations affect the value of the load voltage. For an ideal diode and no ripple, the dc load voltage out of a filtered bridge rectifier equals the peak secondary voltage:

$$V_{dc} = V_{p2}$$

This is what you want to remember when you are trouble-shooting or making a preliminary analysis of a filtered bridge rectifier.

Second Approximation

With the second approximation of a diode, we have to allow for the 0.7 V across each diode. Since there are two conducting diodes in series with the load resistor, the dc load voltage with no ripple out of a filtered bridge rectifier is

$$V_{dc} = V_{p2} - 1.4\text{V}$$

Third Approximation

In the third approximation, two bulk resistances are in the charging path of the capacitor. This complicates the analysis because the diode conducts briefly only near the peak. Fortunately, bulk resistances of rectifier diodes are typically less than 1Ω. Because of this, they usually have little or no effect on the load voltage. Unless you are designing a filtered bridge rectifier, you will not need to consider the effect of bulk resistance. (If you are designing the circuit, you will need to use advanced mathematics because you have to deal with an exponential function. The alternative is to build the circuit and arrive at circuit values by experiment. The main rule here is to keep the load resistance as large as possible compared to the bulk resistance.)

There is one more improvement that we can use. We can include the effect of the ripple as follows:

$$V_{dc(withripple)} = V_{dc(withoutripple)} - \frac{V_R}{2}$$

The idea here is to subtract half the peak-to-peak ripple to refine the answer slightly. Since peak-to-peak is usually less than 10 percent, the improvement in the answer is less than 5 percent.

A Basic Guideline

The resistors used in typical electronic circuits have tolerances of ± 5 percent. Sometimes, you will see precision resistors of ± 1 percent used in critical applications. And sometimes, you will see resistors of ± 10 percent used. But if we take 5 percent as the usual tolerance, then one guideline for selecting an approximation is this: Ignore a quantity if it produces an error of less than 5 percent. This means we can use the ideal diode if it produces less than 5 percent error. If the ideal diode results in 5 percent or more error, switch to the second approximation. Also, ignore the effect of ripple when it is less than 10 percent of the load voltage. (Remember: the peak-to-peak ripple is divided by two before subtracting from the load voltage. Therefore, a 10 percent ripple produces only a 4 percent error in load voltage.)

The foregoing guideline will be of some help in deciding which approximation to use, but don't lean on this guideline too heavily. You may have a situation where a 5 percent guideline is not suitable, Remember the artist's brushes. The job may require a smaller or larger brush. It is impossible to give you a rule for every situation because real life is too messy and has too many exceptions. But don't be discouraged. That's what makes electronics more interesting than accounting. Use the basic guideline given here, but be ready to abandon it if you feel it doesn't apply to your situation.

Example 4-7

Suppose a bridge rectifier has a dc load current of 10 mA and a filter capacitance of 470 μF. What is the peak-to-peak ripple out of a capacitor-input filer?

Solution

Use Eq. (4-8) to get

$$V_R = \frac{10\text{mA}}{(120\text{Hz})(470\mu\text{F})} = 0.117\text{V}$$

This assumes the input frequency is 60 Hz,. which is the normal line frequency in the United States.

Example 4-8

Assume we have a filtered bridge rectifier with a line voltage of 120 V rms, a turns ratio of 9.45, a filter capacitance of 470 μF, and a load resistance of 1 kΩ. What is the dc load voltage?

Solution

Start by calculating the rms secondary voltage:

$$V_2 = \frac{120\text{V}}{9.45} = 12.7\text{V}$$

This is what you would measure with an ac voltmeter connected across the secondary winding.

Next, calculate the peak secondary voltage:

$$V_{p2} = \frac{12.7\text{V}}{0.707} = 18\text{V}$$

With an ideal diode and ignoring the ripple, the dc load voltage equals the peak secondary voltage:

$$V_{dc} = 18\text{V}$$

This answer would be adequate if you were troubleshooting a circuit like this. The dc load voltage is the approximate value you would read with a dc voltmeter across the load resistor. If there were trouble in such a circuit, the dc voltage probably would be much lower than 18 V.

The second approximation improves the answer by including the effect of the two-diode voltage drops:

$$V_{dc} = 18\text{V} - 1.4\text{V} = 16.6\text{V}$$

This is more accurate, so let us use it in the remaining calculations.

To calculate the ripple, we need the value of dc load current:

$$I = \frac{16.6\text{V}}{1k\Omega} = 16.6\text{mA}$$

Now, we can use Eq. (4-8):

$$V_R = \frac{16.6\text{mA}}{(120\text{Hz})(470\mu F)} = 0.294\text{V}$$

This is the peak-to-peak ripple and is what you would see if you looked at the load voltage with the ac input of an oscilloscope. This ripple has little effect on the dc load voltage:

$$V_{dc}\,(\text{with ripple}) = 16.6 - \frac{0.294\ \text{V}}{2} = 16.5\text{V}$$

This gives you the basic idea of how to calculate the dc load voltage and ripple.

Voltage Multipliers

A *voltage multiplier* is two or more peak detectors or peak rectifiers that produce a dc voltage equal to a multiple of the peak input voltage ($2V_p, 3V_p, 4V_p$, and so on). These power supplies are used for high voltage/low current devices like cathode-ray tubes (the picture tubes in TV receivers, oscilloscopes, and computer displays).

Half-Wave Voltage Doubler

Figure 4-15a is a voltage doubler. At the peak of the negative half cycle, D_1 is forward-biased and D_2 is reverse-biased. Ideally, this charges C_1 to the peak voltage, V_p, With the polarity shown in Fig. 4-15b. At the peak of the positive half cycle, D_1 is reverse-biased and D_2 is forward-biased. Because the source and C_1 are in series, C_2 Will try to charge toward $2V_p$. After several cycles, the voltage across C_2 Will equal $2V_p$, as shown in Fig. 4-15c.

By redrawing the circuit and connecting a load resistance, we get Fig. 1-15d. Now it's clear that the final capacitor discharges through the load resistor. As long as R_L is large, the output voltage equals $2V_p$ (ideally). That is, provided the load is light (long time constant), the output voltage is double the peak input voltage. This input voltage normally comes from the secondary winding of a transformer.

For a given transformer, you can get twice as much output voltage as you get from a standard peak rectifier. This is useful when you are trying to produce high voltages (several hundred volts or more). Why? Because higher secondary voltages result in bulkier transformers. At some point, a designer may prefer to use voltage doublers instead of bigger transformers.

The circuit is called a half-wave doubler because the output capacitor, C_2, is charged only once during each cycle. As a result, the ripple frequency is 60 Hz. Sometimes you will see a surge resistor in series with C_1.

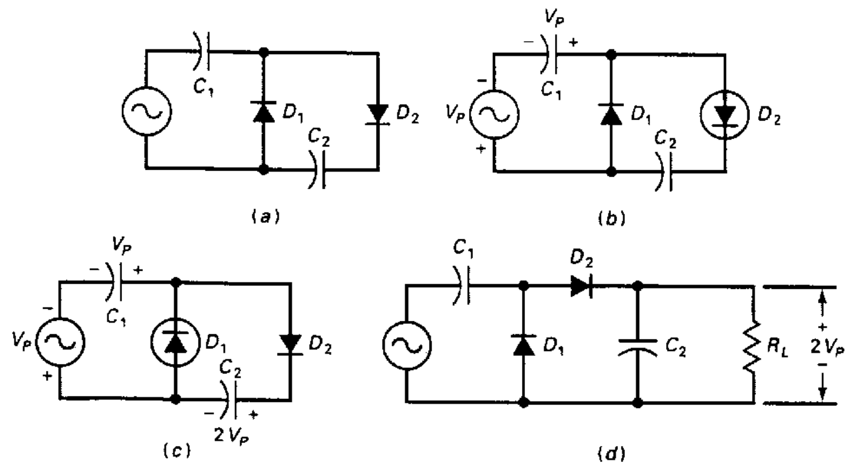


Figure 4-15
Half-wave Voltage Doubler

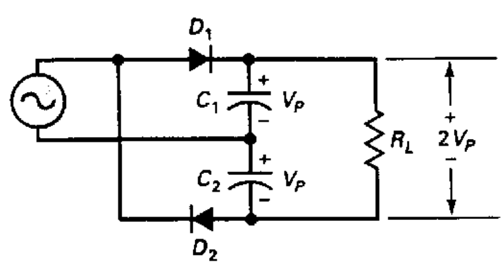


Figure 4-16
Full-wave Voltage Doubler

Full-Wave Voltage Doubler

Figure 4-16 shows a full-wave voltage doubler. On the positive half cycle of the source, the upper capacitor charges to the peak voltage with the polarity shown. On the next half cycle, the lower capacitor charges to the peak voltage with the indicated polarity. For a light load, the final output voltage is approximately $2V_p$.

The circuit is called a full-wave voltage doubler because one of the output capacitors is being charged during each half cycle. Stated another way, the output ripple is 120 Hz. This ripple frequency is an advantage because it is easier to filter. Another advantage of the full-wave doubler is that the PIV rating of the diodes need only be greater than V_p.

The disadvantage of a full-wave doubler is the lack of a common ground between input and output. In other words, if we ground the lower end of the load resistor in Fig. 4-16, the source is Floating. In the half- wave doubler of Fig. 4-15d, grounding the load resistor also grounds the source, an advantage in some applications.

Study Aids

The following study aids will help to reinforce the ideas discussed in this chapter. For best results, use these study aids within 6 hours of reading the earlier material. Then review these study aids a week later and a month later to ensure that the concepts remain in your long-term memory.

Sec. 4-1 The Input Transformer

The input transformer is usually a step-down transformer. In this type of transformer, the voltage is stepped down and the current is stepped up. One way to remember this is by remembering that the output power equals the input power in a lossless transformer.

Sec. 4-2 The Half-wave Rectifier

The half-wave rectifier has a diode in series with a load resistor. The load voltage is a half-wave rectified sine wave with a peak value approximately equal to the peak secondary voltage. The dc or average load voltage equals 31.8 percent of the peak load voltage.

See. 4-3 The Full-wave Rectifier

The full-wave rectifier has a center-tapped transformer with two diodes and a load resistor. The load voltage is a full-wave rectified sine wave with a peak value approximately equal to half of the peak Secondary voltage. The dc or average load voltage equals 63.6 percent of the peak load voltage. The ripple frequency equals two times the input frequency.

See. 44 The Bridge Rectifier

The bridge rectifier has four diodes. The load voltage is a full-wave rectified sine wave with a peak value approximately equal to peak secondary voltage. The de or average load voltage equals 63.6 percent of the peak load voltage. The ripple frequency equals two times the line frequency.

Sec, 4-5 The capacitor-input Filter

This is a capacitor across the load resistor, The idea is to charge the capacitor to the peak voltage and let it supply current to the load when the diodes are nonconducting. With a large capacitor, the ripple is small and the load voltage is almost a pure dc voltage.

See. 4-6 Calculating Other Quantities

In a full-wave or bridge rectifier, the diode current is half the load current and the peak inverse voltage equals the peak secondary voltage. In any kind of rectifier, the primary current approximately equals the load power divided by the primary voltage.

See. 4-7 Surge Current

Because the filter capacitor is uncharged before the power is turned on, the initial charging current is quite high. If the filter capacitor is less than 1000 μF, the surge current is usually too brief to damage the diodes.

See. 4-8 Troubleshoot

The basic measurements you can make on a rectifier circuit include a floating ac voltmeter across the secondary winding to measure the rms secondary voltage, a dc voltmeter across the load resistor to measure the dc load voltage, and an oscilloscope across the load resistor to measure the peak-to-peak ripple.

Sec, 4-9 Reading a Data Sheet

The three most important specifications on the data sheet of a diode are the peak reverse voltage, the maximum diode current, and the maximum surge current.

Vocabulary

In your own words, explain what each of the following terms means . Keep your answers short and to the point. If necessary, verify your answer by rereading the appropriate discussion or by looking at the end-of-book Glossary.

bridge rectifier	peak value
capacitor-input filter	rectifier diode
dc value	ripple
full-wave rectifier	rms value
half-wave rectifier	step-down transformer
line voltage	surge current
peak inverse voltage	

Important Equations

The following formulas are useless if you don't know what they mean in words. Suggestions: Look at each formula, then read the words to find out what the formula means. Your chances of learning and remembering are much better if you concentrate on words rather than formulas:

Eq. 4-1 RMS Voltage

$$V_{rms} = .707V_p$$

This equation relates the heating effect of a dc voltage to an ac voltage. In effect, it converts a sine wave with a peak value of V_p to a dc voltage with a value of V_{rms}. It says a sine wave with a peak value of V_p produces the same amount of heat or power as a dc voltage with a value of V_{rms}. The magic number 0.707 comes from a calculus derivation. There's not much else you can do here except memorize the relation.

Eq. 4-5 DC Voltage from a Half-wave Rectifier

$$V_{dc} = 0.318V_p$$

One of the things you can do with calculus is work out the average value of time-varying signal. If you really want to know where the number 0.318 comes from, you will have to learn calculus. Otherwise, just memorize the equation. It says the dc or average value of a half-wave rectified sine wave equals .318 percent of the peak voltage.

Eq. 4-6 DC Voltage from a Full-wave Rectifier

$$V_{dc} = 0.636\ V_P$$

Because the fill-wave signal has twice as many cycles as a half-wave signal, the average voltage is twice as much. The question says that the dc voltage equals 63.6 percent of the peak voltage of the full-wave rectified sine wave.

Eq. 4-7 DC Frequency from Full-wave Voltage

$$f_{out} = 2f_{in}$$

This applies to full-wave and bridge rectifiers. It says the ripple frequency equals two times the line frequency. If line frequency is 60 Hz, the ripple frequency is 120 Hz. Very important for troubleshooting. Remember it.

Eq. 4-8 DC Ripple out of Capacitor-Input Filter

$$VR = \frac{I}{FC}$$

This equation is the key to the value of ripple, something a troubleshooter or designer needs to know. It says that the peak-to-peak ripple equals the dc load current divided by the ripple frequency times the filter capacitance.

Eq. 4-9 DC Diode Current

$$I_D = 0.5I_L$$

This applies to full-wave and bridge rectifiers. The equation says that the dc current in any diode equals half the dc load current.

Eq. 4-10 DC Peak Inverse Voltage

$$\text{PIV} = V_{p2}$$

This applies to full-wave and bridge rectifiers. It says that the peak inverse voltage across a non conducting diode equals the peak secondary voltage.

Questions

The following may have more than right answer. Select the best answer. This is the one that is always true, or covers more situations, or fits the context, etc.

1. If N1/N2 = 2, and the primary voltage is 120 V, what is the secondary voltage?

   a. 0 Vc. 40 V

   b. 36 V d. 60 V

2. In a step-down transformer, which is larger?

   a. Primary voltage c. Neither

   b. Secondary voltage d. No answer possible

3. A transformer has a turns ratio of 4:1. What is the peak secondary voltage if 115 V rms is applied to the primary winding?

   a. 40.7 V c. 163 V

   b. 64.6 V d. 170 V

4. With a half-wave rectified voltage across the load resistor, load current flows for what part of a cycle?

   a. 0° c. 180°

   b. 90° d. 360°

5. Suppose line voltage may be as low as 105 V rms or as high as 125 rms in a half-wave rectifier. With a 5:1 step-down transformer, the maximum peak load voltage is closest to

   a. 21 V c. 29.6 V

   b. 25 V d. 35.4 V

6. The voltage out of a bridge rectifier is

   a. Half-wave signal b. Full-wave signal

   c. Bridge-rectified signal d. Sinewave

7. If the line voltage is 115 V rms, a turns ratio of 5:1 means the rms secondary voltage is closest to

   a. 15 V c. 30 V

   b. 23 V d. 35 V

8. What is the peak load voltage in a full-wave rectifier if the secondary voltage is 20 V rms?

   a. 0 V c. 14.1 V

   b. 0.7 V d. 28.3 V

9. We want a peak load voltage of 40 V out of a bridge rectifier. What is the approximate rms value of secondary voltage?

   a. 0 V c. 28.3 V

   b. 14.4 V d. 56.6 V

10. With a full-wave rectified voltage across the load resistor, load current flows for what part of a cycle?

    a. 0° c. 180°

    b. 90° d. 360°

11. What is the peak load voltage out of a bridge rectifier for a secondary voltage of 15 V rms? (Use second approximation.)

    a. 9.2 V c. 19.8 V

    b. 15 V d. 24.3 V

12. If line frequency is 60 Hz, the output frequency of a half-wave rectifier is

    a. 30 Hz c. 120 Hz

    b. 60 Hz d. 240 Hz

13. If line frequency is 60 Hz, the output frequency of a bridge rectifier is

    a. 30 Hz c. 120 Hz

    b. 60 Hz d. 240 Hz

14. With the same secondary voltage and filter, which has the most ripple?

    a. Half-wave rectifier c. Bridge rectifier

b. Full-wave rectifier d. Impossible to say

15. With the same secondary voltage and filter, which produces the least load voltage?

a.	Half-wave rectifier	c.	Bridge rectifier
b.	Full-wave rectifier	d.	Impossible to say

16. If the filtered load current is 10 mA, which of the following has a diode current of 10 mA?

a.	Half-wave rectifier	c.	Bridge rectifier
b.	Full-wave rectifier	d.	Impossible to say

17. If the load current is 5 mA and the filter capacitance is 1000 μ F, what is the peak-to-peak ripple out of a bridge rectifier?

a.	21.3 pV	c.	21.3 mV
b.	56.3 nV	d.	41.7 mV

18. The diodes in a bridge rectifier each have a maximum de current rating of 2 A. This means the de load current can have a maximum value of

a.	1 A	c.	4 A
b.	2 A	d.	8 A

19. What is the PIV across each diode of a bridge rectifier with a secondary voltage of 20 V rms?

a.	14.1 V	c.	28.3 V
b.	20 V	d.	34 V

20. If the secondary voltage increases in a bridge rectifier with a capacitor-input filter, the load voltage will

a.	Decrease	c.	Increase
b.	Stay the same	d.	None of these

21. If the filter capacitance is increased, the ripple will

a.	Decrease	c.	Increase
b.	Stay the same	d.	None of these

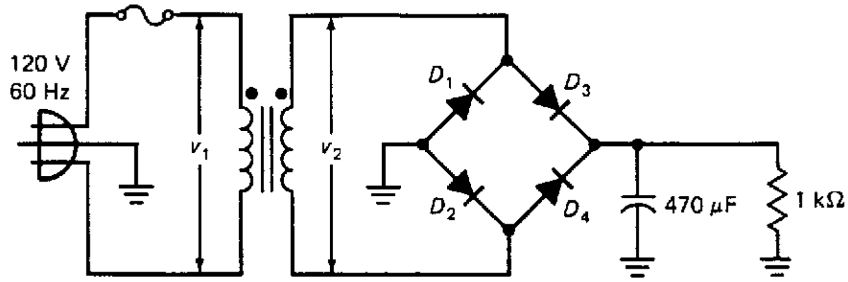


Figure 4-30

22. In Fig. 4-30, the filter capacitor is open. What will the load voltage look like on an oscilloscope?
    a. Horizontal line at 0 V
    b. Horizontal line at normal output
    c. Half-wave signal
    d. Full-wave signal
23. Something is shorting out the load resistor of Fig. 4-30. After you remove the short, you should check the condition of the
    a. Fuse
    b. Odd-numbered diodes
    c. Even-numbered diodes
    d. All of the foregoing
24. In Fig. 4-30, the secondary voltage has an rms value of 12.7 V. If a dc voltmeter indicates a load voltage of 11.4 V, the trouble is probably
    a. An open filter capacitor
    b. Blown fuse
    c. Open secondary winding
    d. No center tap
25. The dc load voltage of Fig. 4-30 seems normal, but the ripple is 60 Hz. Which of these is a possible trouble:
    a. An open filter capacitor
    b. Blown fuse
    c. Open secondary winding
    d. Open diode

Sec. 4-1 The Input Transformer

4-1. Suppose the peak value of a sinusoidal voltage is 50 V. What is the rms value?

4-2. Line voltage may vary from 105 to 125 V rms. Calculate the peak value for low-line voltage and high-line voltage.

4-3. A step-up transformer has a turns ratio of 1:4. If the line voltage is 115 V rms, what is the peak secondary voltage?

4-4. A step-down transformer has a primary voltage of 110 V rms and a secondary voltage of 12.7 V rms. What is the turns ratio?

4-5. A transformer has a primary voltage of 120 V rms and a secondary voltage of 25 V rms. If the secondary current is 1A rms, what is the primary current?

Sec 4-2 The Half-wave Rectifier

4-6. During the day the line frequency varies slightly from its nominal value of 60 Hz. Suppose the line frequency is 61 Hz. What is the period of the rectified output voltage from a half-wave rectifier?

4-7. A step-down transformer with a turns ratio of 3:1 is connected to a half-wave rectifier. If the line voltage is 115 V rms, what is the peak load voltage? Give the two answers: one for an ideal diode, and another for the second approximation.

Sec. 4-3 The Full-wave Rectifier

4-8. During the day, the line frequency drops down to 59 Hz. What is the frequency out of a full-wave rectifier for this input frequency? What is the period of the output?

4-9. Refer to Fig. 4-7. Suppose the line voltage varies from 105 V rms to 125 V rms. What is the peak load voltage for the two extremes? (Use ideal diodes.)

4-10. If the turns ratio of Fig. 4-7 is changed to 6:1, what is the dc load current?

Sec. 4-4 The Bridge Rectifier

4-11. Refer to Fig. 4-10. If the load resistance is changed to 3.3 kΩ, what is the dc load current? Give answers for two cases: ideal diode and second approximation.

4-12. If in Fig. 4-10, the turns ratio is changed to 6:1 and the load resistance to 820Ω, what is the dc load current? (Give ideal- and second-approximation answers.)

Sec. 4-5 The Capacitor-input Filter

4-13. A bridge rectifier has a dc load current of 20 mA and a filter capacitance of 680 ΩF. What is the peak-to-peak ripple out of a capacitor-input filter?

4-14. In the previous problem, the rms secondary voltage is 15 V. What is the dc load voltage? Give three answers: one based on ideal diodes, another based on the second approximation, and a third based on the effect of ripple.

Sec. 4-6 Calculating Other Quantities

4-15. The rms secondary voltage of Fig. 4-30 is 12.7 V. Use the ideal diode and ignore the effect of ripple on dc load voltage. Work out the values of each of these quantities: dc load voltage, dc load current, dc diode current, rms primary current, peak inverse voltage, and turns ratio.

4-16. Repeat Prob. 4-15, but use the second approximation and include the effect of ripple on the dc load voltage.

4-17. Draw the schematic diagram of a bridge rectifier with a capacitor-input filter and these circuit values: V2 = 20 V, C 1000 Ω *F, RL* = 1 kΩ. What is the load voltage and peak-to-peak ripple?

Sec. 4-8 Troubleshooting

4-18. You measure 24 V rms across the secondary of Fig. 4-30. Next you measure 21.6 V dc across the load resistor. What is the most likely trouble?

4-19. The dc load voltage of Fig. 4-30 is too low. Looking at the ripple with a scope, you discover it has a frequency of 60 Hz. Give some possible causes.

4-20. There is no voltage out of the circuit of Fig. 4-30. Give some possible troubles.

4-21. Checking with an ohmmeter, you find all diodes in Fig. 4-30 open. You replace the diodes. What else should you check before you power up?

Advanced Problems

4-22 You are designing a bridge rectifier with a capacitor-input filter. The specifications are a dc load voltage of I5 V and a ripple of 1 V for a load resistance of 680 Ω. How much rms voltage should the secondary winding produce for a line voltage of 15 V rms? What size should the filter capacitor be? What are the minimum I_o and PIV ratings for diodes?

4-23. Design a full-wave rectifier using a 48 V rms center-tapped transformer that produces a 10 percent ripple across a capacitor-input filter with a load resistance of 330 Ω. What are the minimum I_o and PIV ratings of the diodes?

4-24. Design a power supply to meet the following specifications: The secondary voltage is 12.6 V rms and the dc output is approximately 17.8 V at 120 mA. What are the minimum I_o and PIV ratings of the diodes?

4-25 A full-wave signal has a dc value of 0.636 times the peak value. With your calculator or a table of sine values, you can derive the average value of 0.636. Describe how you would do it.

4-26. The secondary voltage in Fig. 4-31 is 25 V rms. With the switch in the upper position, what is the output voltage?

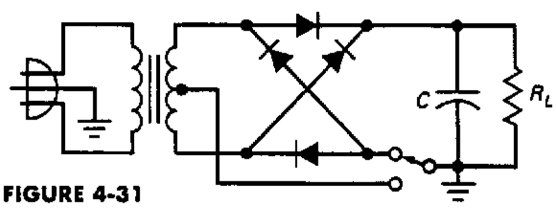
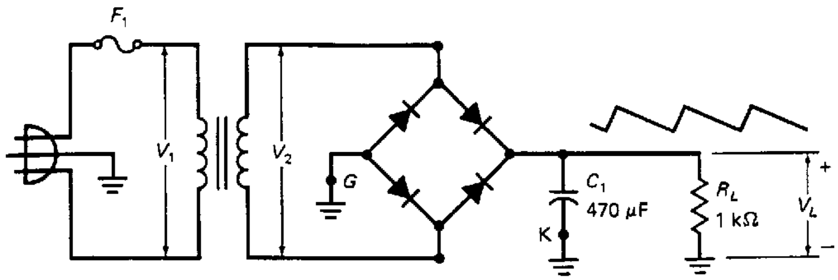


FIGURE 4-31

4-27. A rectifier diode has a forward voltage of 1.2 V at 2 A. The winding resistance is 0.3 Ω. If the secondary voltage is 25 V rms, what is the surge current in a bridge rectifier?

Use Fig. 4-32 for the remaining problems. If you haven't already done so, read Example 4-12 before attempting these problems. You can measure voltages in any order; for instance, V_2 first, V_L second, and V_R third, or whatever. These voltages are the clues to the trouble. After measuring a voltage, try to figure out what to measure next. Troubleshooting has so many possibilities that it is impractical to try to give rules for every situation. The best approach is to measure something, then think about what this tells you. Usually, the measurement gives you an idea of what you should measure next. Keep making measurements until you have enough clues to logically figure out what the trouble is.

The possible troubles are open or shorted components (diodes, resistors, capacitors, etc.). Besides voltage measurements, there are other measurements as follows: f for ripple frequency, R_L for load resistance, C_1 for capacitor resistance, and F_l for fuse resistance.



	OK	T 1	T 2	T 3	T 4	T 5	T 6	T 7	T 8	T 9
V_1	D2	F3	A1	C1	D4	F4	A1	D2	A4	D4
V_2	B6	B2	C4	A4	E2	E7	E2	C1	C1	B6
V_L	F5	D7	F2	A7	G5	A2	F5	A6	F4	D7
V_R	G1	E1	D6	B5	A6	F6	A7	E7	A7	E1
f	A3	C6	G4	C3	G2	G7	F6	G2	G5	C6
R_L	C5	E4	A5	D1	F1	C2	B3	C5	E4	C2
C_1	F7	G3	D3	C7	E3	B1	D5	F7	B5	B7
F_1	B4	B7	E6	E5	D5	B3	B4	B4	B3	D3

	1	2	3	4	5	6	7
A	115	0	120	0	1 k	0	0
B	OK	12.7	∞	OK	0	12.7	OK
C	0	1 k	0	12.7	1 k	120	OK
D	0	115	OK	115	OK	0.6	11.4
E	18	12.7	OK	1 k	∞	OK	0
F	1 k	17.7	115	0	18	0	OK
G	0.3	0	∞	60	0	120	0

MEASUREMENTS

FIGURE 4-32 T-Shooter™. (*Patent pending: Courtesy of Malvino Inc.*)

4-28. Find Trouble 1.

4-29. Find Troubles 2 and 3.

4-30. Find Troubles 4 and 5.

4-31. Find Troubles 6 and 7.

4-32. Find Troubles 8 and 9.

4-1. 35.4 V

4-3. 651 V

4-5. 208 mA

4-7. 54.2 V and 53.5 V

4-9. 14.9 V and 17.7 V

4.11. 6.54 mA (ideal) and 6.27 mA (second)

4.13. 0.245 V

4.15. 18 V, 18 mA, 9 mA, 2.7 mA, 18 V, and 9.45

4.17. Ideal: 28.3 V and 0.236 V; second: 26.9 V and 0.224 V

4.19. Possible troubles include and open diode or an open connection in one of the diode branches.

4.21. You should check the load resistance to see if it is being shorted out.

4.23. Ideal and ignore ripple. V_L = 33.9 V, C = 252 μF, I_O = 51 mA, and PIV = 33.9; second and ignore ripple: . V_L = 32.5 V, C = 252 μF, I_O = 49.2 mA, and PIV = 33.9; second and include ripple: V_L = 30.9 V, C = 252 μF, I_O = 46.8 mA, and PIV = 33.9 V

4.25. We can look up the sine of the angle every 5 degrees between 0° and 90°. There are 19 samples including the sine of 0°. By adding up the sine values and dividing by 19, we get 0.629. This is close to the exact value of 0.636. If a more accurate answer is needed, we could use a smaller interval, say every degree.

4.27. 44.2 A

4-29. Trouble 2: Diode open; Trouble 3: Load resistor shorted

4-31. Trouble 6: Load resistor open; Trouble 7: Secondary winding open.

Special Purpose Diodes

Rectifier diodes are the most common type of diode. They are used in power supplies to convert ac voltage to dc voltage. But rectification is not all that a diode can do. Now we will discuss diodes used in other applications. The chapter begins with the zener diode, which is optimized for its breakdown properties. Zener diodes are very important because they are the key to voltage regulation. The chapter also covers optoelectronic diodes. Schottky diodes, varactors, and other diode

The Zener Diode

Small-signal and rectifier diodes are never intentionally operated in the breakdown region because this may damage them. A zener diode is different; it is a silicon diode that the manufacturer has optimized for operation in the breakdown region. In other words, unlike ordinary diodes that never work in the breakdown region, zener diodes work best in the breakdown region. Sometimes called a breakdown diode, the zener diode is the backbone of voltage regulators, circuits that hold the load voltage almost constant despite large changes in line voltage and load resistance.

I-V Graph

Figure 5-l*a* shows the schematic symbol of a zener diode; Fig. 5 -1*b* is an alternative symbol. In either symbol, the lines resemble a "z," which stands for zener. By varying the doping level of silicon diodes, a manufacturer can produce zener diodes with breakdown voltages from about 2 to 200 V. These diodes can operate in any of three regions: forward, leakage, and breakdown.

Figure 5-1*c* shows the I-V graph of a zener diode. In the forward region, it starts conducting around 0.7 V, just like an ordinary silicon diode. In the leakage region (between zero and breakdown) it has only a small reverse current. In a zener diode, the breakdown has a very sharp knee, followed by an almost vertical increase in current. Note that the voltage is almost constant, approximately equal to V_Z over most of the breakdown region. Data sheets usually specify the value of V_Z at a particular test current I_{ZT}.

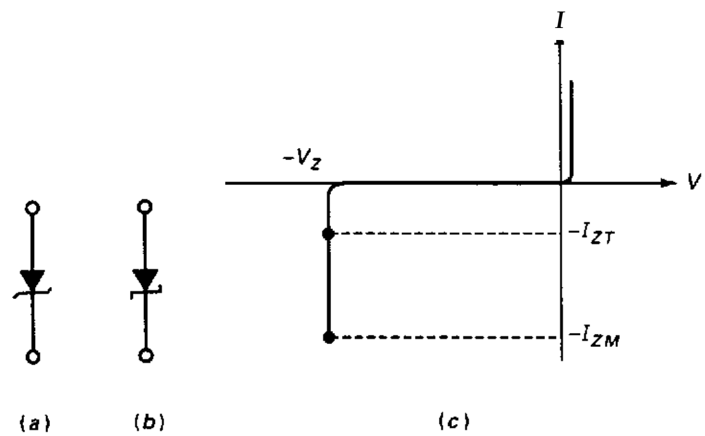


Figure 5-1
Zener Diode
(a) Symbol (b) Alternative Symbol (c) Diode Curve

Do not let the use of the minus signs confuse you. Minus signs need to be included with graphs because you are simultaneously showing forward and reverse values. But you don't have to use minus signs in other discussions if the meaning is clear without them. For instance, it is preferable to say that a zener diode has a breakdown voltage of 10 V, rather than to say it has a breakdown voltage of - 10 V. Anyone who knows how a zener diode works already knows it has to be reverse-biased. A pure mathematician might prefer to say a zener diode has a breakdown voltage of - 10 V, but a practicing engineer or technician will prefer to say it has a breakdown voltage of 10 V.

Zener Resistance

Because all diodes have some bulk resistance in the *p* and *n* regions, the current through a zener diode produces a small voltage drop in addition to the breakdown voltage. To state it another way, when a zener diode is operating in the breakdown region of Fig. 5-l*c*, an increase in current produces a slight increase in voltage. The increase is very small, typically a few tenths of a volt. This may be important in design work, but not for troubleshooting and preliminary analysis. Unless otherwise indicated, our discussions will ignore the zener resistance.

Zener Regulator

A zener diode is sometimes called a *voltage-regulator* diode because it maintains a constant output voltage even though the current through it changes. For normal operation, you have to reverse-bias the zener diode as shown in Fig. 5-2*a*. Furthermore, to get breakdown operation, the source voltage Vs must be greater than the zener breakdown voltage V_Z. A series resistor R_S is always used to limit the zener current to less than its maximum current rating. Otherwise, the zener diode will burn out like any device with too much power dissipation.

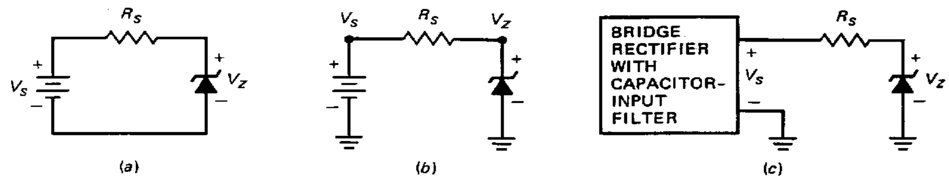


Figure 5-2
Zener Regulator

Figure 5-2*b* shows an alternative way to draw the circuit with grounds. Whenever a circuit has grounds, it is usually best to measure node voltages with respect to ground. In fact, if you are using a voltmeter with a power plug, its common terminal may be grounded. In this case, it is necessary to measure node voltages to ground.

For instance, suppose you want to know the voltage across the series resistor of Fig. 5-2*b.* Here is the usual way to find it when you have a built-up circuit. First, measure the voltage from the left end of R_S to ground. Second, measure the voltage from the right end of R_S to ground. Third, subtract the two voltages to get the voltage across R_S. This indirect method is necessary because the common lead of many plug-in voltmeters is grounded. (Note: If you have a floating VOM, you can connect directly across the series resistor.)

Figure 5-2*c* shows the output of a power supply connected to a series resistor and a zener diode. This circuit is used when you want a de output voltage that is less than the output of the power supply. A circuit like this is called a *zener voltage regulator*, or simply a *zener regulator*.

Ohm's Law Again

In Fig. 5-2, the voltage across the series resistor equals the difference between the source voltage and the zener voltage. Therefore, the current through the resistor is

$$I_S = \frac{V_S - V_Z}{R_S} \qquad (5\text{-}1)$$

Don't memorize this equation. It is nothing more than Ohm's law applied to the series resistor. The series current equals the voltage across the series resistor divided by the resistance. The only thing you have to remember is that the voltage across the series resistor is the difference between the source voltage and the zener voltage. In fact, you don't even have to remember that because the circuit itself contains this information. When you look at Fig. 5-2, you can see at a glance that the voltage across the series resistor equals V_S minus V_Z.

Once you have the value of series current, you also have the value of zener current. Why? Because Fig. 5-2 is a series circuit and you know that current is the same in all parts of a series circuit.

Ideal Zener Diode

For troubleshooting and preliminary analysis, we can approxi-mate the breakdown region as vertical. Therefore, the voltage is constant even though the current changes, which is equivalent to ignoring the zener resistance. Figure 5-3*a* shows the ideal approximation of a zener diode. This means that a zener diode operating in the breakdown region ideally acts like a battery. In a circuit, it means that you can mentally replace a zener diode by a voltage source of V_Z, provided the zener diode is operating in the breakdown region.

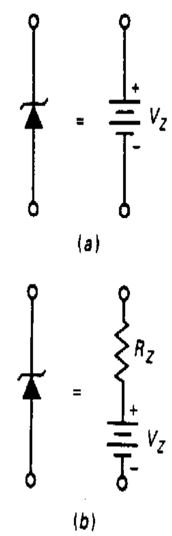


Figure 5-3
Zener approximation
(a) Ideal; (b) Second approximation

Second Approximation

Figure 5-3*b* shows the second approxima-tion of a zener diode. A zener resistance (relatively small) is in series with an ideal battery. This resistance produces a voltage drop equal to the product of the current and the resistance.

Example 5-1

Suppose the zener diode of Fig. 5-4a has a breakdown voltage of 10V. What are the minimum and maximum zener currents?

Solution

The applied voltage may vary from 20 to 40 V. Ideally, a zener diode acts like the battery shown in Fig. 5-4b, Therefore, the output voltage is 10 V for any source voltage between 20 and 40 V.

The minimum current occurs when the source voltage is minimum. Visualize 20 V on the left end of the resistor and 10 V on the right end. Then you can see that the voltage across resister is 20 V - 10 V, or 10 V. The rest is Ohm's law:

$$I_S = \frac{10\text{V}}{820\Omega} = 12.2\text{mA}$$

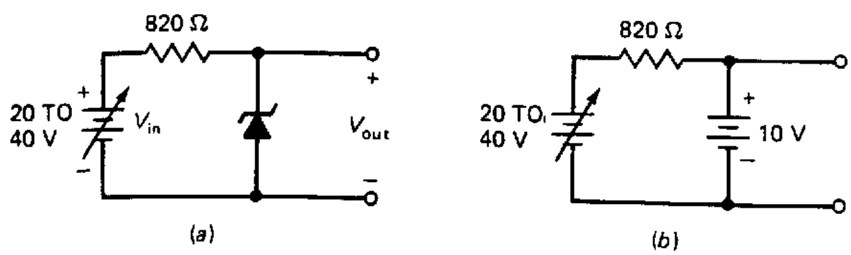


Figure 5–4
Example

The maximum current occurs when the source voltage is 40 V. In this case, the voltage across resistor is 30 V, which gives a current of

$$I_S = \frac{30\text{V}}{820\Omega} - 36.6\text{mA}$$

In a voltage regulator like Fig. 5-4*a*, the output voltage is held constant at 10 V, despite the change in source voltage from 20 to 40 V. The larger source voltage produces more zener current, but the output voltage holds rock-solid at 10 V. (If the zener resistance is included, the output voltage increases slightly when the source voltage increases.)

Figure 5-5*a* shows a loaded zener regulator, and Fig, 5-5*b* shows the same circuit in a practical form. This circuit is more complicated than the unloaded zener regulator analyzed in the previous section, but the basic idea is the same. The zener diode operates in the breakdown region and holds the load voltage constant. Even if the source voltage changes or the load resistance varies, the load voltage will remain fixed and equal to the zener voltage.

Breakdown Operation

Always remember this: The zener diode has to operate in the breakdown region to hold the load voltage constant, To put it another way, the zener diode cannot regulate if the load voltage is less than the zener voltage.

How can you tell if the zener diode of Fig, 5-5 is operating in the breakdown region? The designer of the circuit usually takes care of this. Here is the formula that applies:

$$V_{TH} = \frac{R_L}{R_S = R_L} V_S \qquad (5\text{-}2)$$

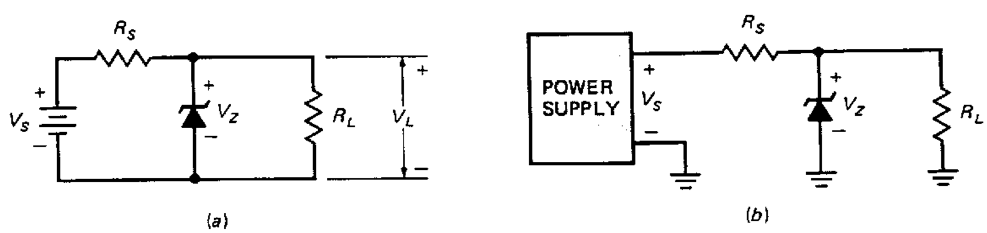


Figure 5–5
Zener Regulator

This is the voltage that exists when the zener diode is disconnected from the circuit. This voltage has to be greater than the zener voltage; otherwise, breakdown cannot occur.

Here is where the equation comes from. When the zener diode is disconnected from the circuit, all that's left is a voltage divider consisting of R_S in series with R_L. The current through this voltage divider is

$$I_S = \frac{V_S}{R_S = R_L}$$

The load voltage without the zener diode equals the previous current times the load resistance. When you multiply the current by the load resistance, you get the right side of Eq. (5-2), where V_{TH} stands for the Thevenin voltage. This is the voltage with the zener diode out of the circuit.

Series Current

Unless otherwise indicated, in all subsequent discussions we assume the zener diode is operating in the breakdown region. In Fig. 5-5, the current through the series resistor is given by

$$I_S = \frac{V_S - V_Z}{R_S} \qquad (5\text{-}3)$$

This is Ohm's law applied to the current-limiting resistor. It is the same whether or not there is a load resistor. In other words, if you disconnect the load resistor, the current through the series resistor still equals the voltage across the resistor divided by the resistance.

Load Current

Ideally, the load voltage equals the zener voltage because the load resistor is in parallel with the zener diode. As an equation,

$$V_L = V_Z \qquad (5\text{-}4)$$

This allows us to use Ohm's law to calculate the load current:

$$I_L = \frac{V_L}{R_L} \qquad (5\text{-}5)$$

Zener Current

With KirchhoffÕs current law,

$$I_S = I_Z + I_L$$

This should be clear from your study of series-parallel circuits. The zener diode and the load resistor are in parallel. The sum of their currents has to equal the total current, which is the same as the current through the series resistor.

We can rearrange the foregoing equation to get this important formula:

$$I_Z = I_S - I_L \qquad (5\text{-}6)$$

This tells you that the zener current no longer equals the series current, as it does in an unloaded zener regulator. Because of the load resistor, the zener current now equals the series current minus the load current.

Process

Troubleshooters, designers, and other professionals don't blindly plug numbers into formulas, hoping to get the right answer. Professionals know the meaning of each step they take when they solve a problem. Knowing what you are doing is a lot better than relying on formulas.

If professionals don't use formulas, what do they use? Some-thing called a *process*. A process is a step-by-step routine used to solve problems. When professionals solve a problem, they work out the values of different quantities, using Ohm's law in a logical sequence. Occasionally, a complicated formula may be necessary, but that is the exception rather than the rule. Often, problems in electronics are simply Ohm's law and other basic ideas applied over and over to the different components and devices in the circuit.

Here is a three-step process for finding the zener current:

1. Calculate the current through the series resistor.
2. Calculate the load current.
3. Calculate the zener current.

These steps can be abbreviated to

1. Series current
2. Load current
3. Zener current

or symbolically,

1. I_S

2. I_L
3. I_Z

This is what professionals remember. You get the series current first, the load current second, and the zener current third. And you use Ohm's and other basic ideas in the process. The details of the calculations are automatically remembered, at least most of the time.

If you can remember the three quantities in the process, your mind usually takes care of the rest of the details. If you do get stuck, look at the formulas to jog your memory. But don't use formulas blindly. Reread the discussion or examples if you can't remember the details of some step in the process. In general, don't memorize any formula unless you expect to use it a few thousand times. Ohm's law is an example of a formula to memorize. The equations of this chapter are examples of formulas you do not memorize because most of them are rewrites of Ohm's law.

Ripple across the Load Resistor

In Fig. 5-5*b*, the output of a power supply drives a zener regulator. As you know, the power supply produces a dc voltage with a ripple. Ideally, the zener regulator reduces the ripple to zero because the load voltage is constant and equal to the zener voltage. As an example, suppose the power supply produces a dc voltage of 20 V with a peak-to-peak ripple of 2 V. Then the supply voltage is swinging from 19 V minimum to 21 V maximum. Variations in supply voltage will change the zener current, out they have almost no effect on the load voltage.

If you take into account the small zener resistance, you will find that there is a small ripple across the load resistor. But this ripple is much smaller than the original ripple coming out of the power supply. In fact, you can estimate the new ripple with this equation:

$$V_{R(out)} = \frac{R_Z}{R_S + R_Z} V_{R(in)} \qquad (5\text{-}7)$$

This is an accurate approximation of peak-to-peak output ripple. If it reminds you of a voltage divider, you are right on target. It comes from visualizing the zener diode replaced by its second approximation. With respect to the ripple, the circuit acts like a voltage divider formed by R_S in series with R_Z.

Temperature Coefficient

One final point: Raising the ambient (surrounding) temperature changes the zener voltage slightly. On data sheers, the effect of temperature is listed under the temperature coefficient, which is the percentage change per degree change. A designer needs to calculate the change in zener voltage at the highest ambient temperature. But even a troubleshooter should know that temperature can change the zener voltage.

For zener diodes with breakdown voltages less than 5 V, the temperature coefficient is negative. For tenet diodes with breakdown voltages of more than 6 V, the temperature coefficient is positive. Between 5 and 6 V, the temperature coefficient changes from negative to positive; this means that you can find an operating point for a zener diode at which the temperature coefficient is zero. This is important in some applications where a solid zener voltage is needed over a large temperature range.

Example 5-2

Figure 5-6 has these circuit values: V_S = 18 V, V_Z 10 V, R_S = 270 Ω, and R_L = 1kΩ. Is the zener diode operating in breakdown region?

Solution

Use Eq. (5-2), or better still, use your head. Mentally disconnect the zener diode. Then all that is left is a voltage divider with 270 Ω in series with 1kΩ. Therefore, the current through the voltage divider is

$$I = \frac{18\text{V}}{1.27k\Omega} = 14.2\text{mA}$$

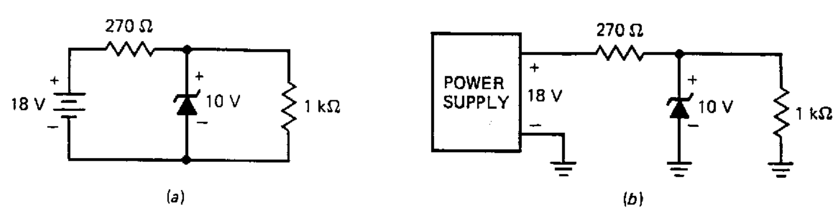


Figure 5–6
Example

Multiply this current by the total resistance to get the Thevenin voltage :

$$V_{TH} = (14.2 \text{ mA})(1 \text{ k}\Omega) = 14.2 \text{ V}$$

Since this voltage is greater than the zener voltage (10 V), the zener diode will operate in the breakdown region when it is reconnected to the circuit.

Naturally, you can plug the values directly into Eq. (5-2) as follows:

$$V_{TH} = \frac{1K\Omega}{1.27K\Omega} 18\text{V} = 14.2\text{V}$$

The result is the same, so either method is acceptable. The advantage of the first method is that you are more likely to remember it because it is Ohm's law applied twice. Also, the first method requires you to think logically about what is happening in the circuit. But either method is valid, so use whichever you prefer.

Example 5-3

What does the zener current equal in Fig. 5-6*b*?

Solution

You are given the voltage on both ends of the series resistor. Subtract the voltages, and you can see that 8 V is across the series resistor. Then Ohm's law gives

$$I_S = \frac{8\text{V}}{270\Omega} = 29.6\text{mA}$$

Since the load voltage is 10 V, the load current is

$$I_L = \frac{10\text{V}}{1k\Omega} = 10\text{mA}$$

The zener current is the difference of the two currents:

$$I_Z = 29.6 \text{ mA} - 10 \text{ mA} = 19.6 \text{ mA}$$

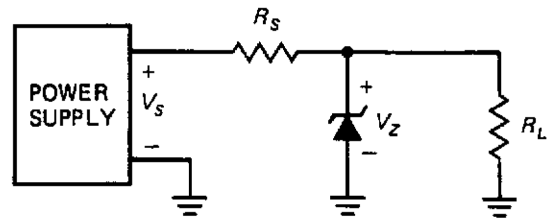


Figure 5–7
Zener Regulator with the Load Resistor

Example 5–4

The data sheet of a 1N961 gives a zener resistance of 8.5 Ω. Suppose this zener diode is used in Fig, 5-7 with a series resistance of 270 Ω.. What is the load ripple if the supply ripple is 2.V?

With Eq. (5-7),

$$V_{R(OUT)} = \frac{8.5\Omega}{278.5\Omega}(2\text{V}) = 0.061\text{V} = 61\text{mV}$$

The final output is a dc voltage of 10V with a peak-to-peak ripple of only 6l mV

Example 5–5

What does the circuit of Fig. 5-8 do?

Solution

This is an example of a preregulator (the first zener diode) driving a zener regulator (the second zener diode). First, notice that the preregulator has an output voltage of 20V. This is the input to the second zener regulator, whose, output is 10 V. The basic idea is to provide the second regulator with a well-regulated input, so that the final output is extremely well regulated.

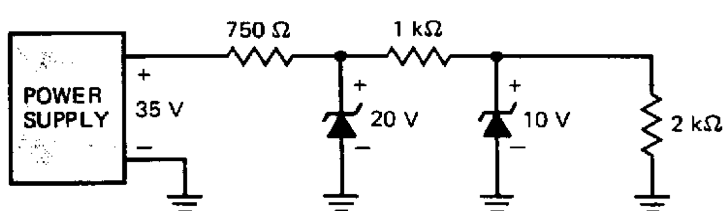


Figure 5–8
Example

Example 5–6

What does the circuit of Fig, 5-9 do?

Solution

In most applications, zener diodes are used in voltage regulators where they remain in the breakdown region. But there are exceptions. Sometimes zener diodes are used in wave shaping circuits like Fig. 5-9.

Notice the back-to-back action of two zener diodes.: On the positive half-cycle, the upper diode conducts and the lower diode breaks down. Therefore, the output is clipped as shown. The clipping level equals the zener voltage (broken-down diode) plus 0.7 V (forward-biased diode). On the negative half-cycle, the action is reversed. The lower diode conducts, and the upper diode breaks down. In this way, the output is almost a square wave. The larger the input sine wave, the better looking the output square wave.

Optoelectronic Devices

Optoelectronics is the technology that combines optics and electronics. This exciting field includes many devices based on the action of a *pn* junction. Examples of optoelectronic devices are light-emitting diodes (LEDs), photodiodes, optocouplers, etc. Our discussion begins with the LED.

Light-Emitting Diode

Figure 5-10*a* shows a source connected to a resistor and a LED. The outward arrows symbolize the radiated light. In a forward-biased LED, free electrons cross the junction and fall into holes. As these electrons fall from a higher to a lower energy level, they radiate energy. In ordinary diodes, this energy goes off in the form of heat. But in a LED, the energy is radiated as light. LEDs have replaced incandescent lamps in many applications because of their low voltage, long life, and fast on-off switching.

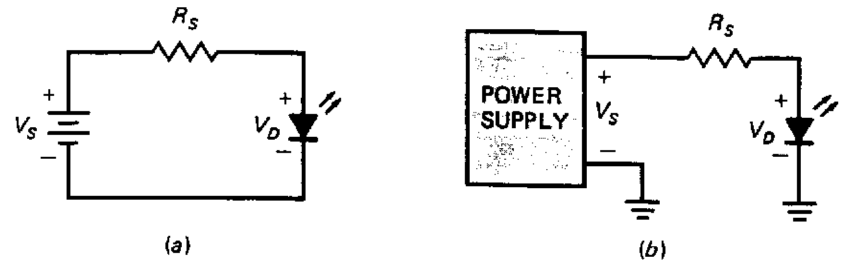


Figure 5–10
LED Circuits

Ordinary diodes are made of silicon, an opaque material that blocks the passage of light. LEDs are different. By using elements like gallium, arsenic, and phosphorus, a manufacturer can produce LEDs that radiate red, green, yellow, blue, orange, or infrared (invisible). LEDs that produce visible radiation are useful with instruments, calculators, etc. The infrared LED finds applications in burglar alarm systems and other areas requiring invisible radiation.

LED Voltage and Current

The resistor of Fig. 5-10 is the usual current-limiting resistor that prevents the current from exceeding the maximum current rating of the diode. Since the resistor has a node voltage of V_S on the left and a node voltage of V_D on the right, the voltage across the resistor is the difference between the two voltages. With Ohm's law, the series current is

$$I_S = \frac{V_S - V_D}{R_S} \qquad (5\text{-}8)$$

For most of the commercially available LEDs, the typical voltage drop is from 1.5 to 2.5 V for currents between 10 and 50 mA. The exact voltage drop depends on the LED current, color, tolerance, etc. Unless otherwise specified, we will use a nominal drop of 2 V when troubleshooting or analyzing the LED circuits in this book. If you get into design work, consult the data sheets for the LEDs you are using.

Seven-Segment Display

Figure 5-11*a* shows a *seven-segment* display. It contains seven rectangular LEDs (*A* through *G*). Each LED is called a segment because it forms part of the character being displayed. Figure 5-11*b* is a schematic diagram of the seven-segment display. External series resistors are included to limit the currents to safe levels. By grounding one or more resistors, we can form any digit from 0 through 9. For instance, by grounding A, B, and C, we get a 7. Grounding A, B, C, D, and G produces a 3.

A seven-segment display can also display capital letters *A, C, E,* and *F*, plus lowercase letters *b* and *d*. Microprocessor trainers often use seven-segment displays that show all digits from 0 through 9, plus *A, b, C, d, E*, and *F.*

The seven-segment indicator of Fig. 5-11*b* is referred to as the common-anode type because all anodes are connected together. Also available is the common-cathode type where all cathodes are connected together.

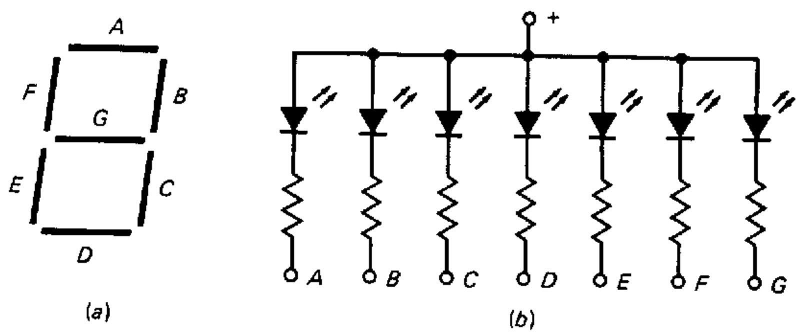


Figure 5–11
(a) Seven-segment Indicator; (b) Schematic Diagram

Photodiode

As previously discussed, one component of reverse current in a diode is the flow of minority carriers. These carriers exist because thermal energy keeps dislodging valence electrons from their orbits, producing free electrons and holes in the process. The lifetime of the minority carriers is short, but while they exist they can contribute to the reverse current.

When light energy bombards a *pn* junction, it can dislodge valence electrons. The more light striking the junction, the larger the reverse current in a diode. A *photodiode* is one that has been optimized for its sensitivity to light. In this diode, a window lets light pass through the package to the junction. The incoming light produces free electrons and holes. The stronger the light, the greater the number of minority carriers and the larger the reverse current.

Figure 5-12 shows the schematic symbol of a photodiode. The arrows represent the incoming Light. Especially important, the source and the series resistor reverse-bias the photodiode. As the light becomes brighter, the reverse current increases. With typical photodiodes, the reverse current is in the tens of microamperes.

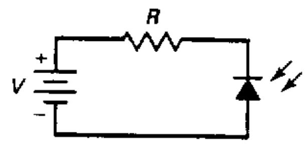


Figure 5–12
Photodiode

Optocoupler

An optocoupler (also called an *optoisolator or an optically coupled isolator*) combined a LED and a photodiode in a single package. Figure 5-13 shows an optocoupler. It has a LED on the input side and a photodiode on the output side. The left source voltage and the series resistor set up a current through the LED. Then the light from the LED hits the photodiode, and this sets up a reverse current in the output circuit. This reverse current produces a voltage across the output resistor. The output voltage then equals the output supply voltage minus the voltage across the resistor.

When the input voltage is varying, the amount of light is fluctuating. This means that the output voltage is varying in step with the input voltage. This is why the combination of a LED and a photodiode is called an optocoupler. The device can couple an input signal to the output circuit.

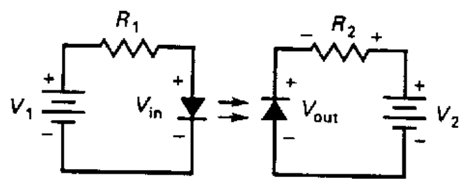


Figure 5–13
Optocoupler

The key advantage of an optocoupler is the electrical isolation between the input and output circuits. With an optocoupler, the only contact between the input and the output is a beam of light. Because of this, it is possible to have an insulation resistance between the two circuits in the thousands of megohms. Isolation like this comes in handy in high-voltage applications where the potentials of the two circuits may differ by several thousand volts.

Example 5–7

In Fig. 5-10 the source voltage is 10 V, and the series resistance is 680 Ω, What is the LED current?

Solution

Use a nominal LED drop of 2 V. Then the series resistor has 10 V on the left end and 2.V on the right end. This means the voltage across the resistor is 8 V. Finish off the problem with Ohm's law:

$$I = \frac{8\text{V}}{680\Omega} = 11.8\text{mA}$$

The Schottky Diode

At lower frequencies, an ordinary diode can easily turn off when the bias changes from forward to reverse. But as the frequency increases, the diode reaches a point where it cannot turn off fast enough to prevent noticeable current during part of the reverse half-cycle. This effect is known as charge storage. It places a limit on the useful frequency of ordinary rectifier diodes.

What happens is this. When a diode is forward-biased, some of the carriers in the depletion layers have not yet recombined. If the diode is suddenly reverse-biased, these carriers can how in the reverse direction for a little while. The greater the lifetime, the longer these charges can contribute to reverse current.

The time it takes to turn off a forward-biased diode is called the reverse recovery time, The reverse recovery time is so short in small-signal diodes that you don't even notice its effect at frequencies below 10 MHz or so. It's only when you get well above 10 MHz that it becomes important.

The solution is a special-purpose device called a Schottky diode. This type of diode has no depletion layer, which eliminates the stored charges at the junction. The lack of charge storage means the Schottky diode can switch off faster than an ordinary diode. In fact, a Schottky diode can easily rectify frequencies above 300 MHz.

The most important application of Schottky diodes is in digital computers. The speed of computers depends on how fast their diodes and transistors can turn on and off. This is where the Schottky diode comes in. Because it has no charge storage, the Schottky diode has become the backbone of low-power Schottky TTL, a group of widely used digital devices.

A final point: In the forward direction, a Schottky diode has a barrier potential of only 0.25 V. Therefore, you may see Schottky diodes used in a low-voltage bridge rectifiers because you have to subtract only 0.25 instead of the usual 0.7 V for each diode.

The Varactor

The varactor (also called the voltage-variable capacitance, varicap, epicap, and tuning diode) is widely used in television receivers, FM receivers, and other communications equipment. Here is the basic idea. In Fig. 5-14*a*, the depletion layer is between the *p* region and the *n* region. The p and n regions are like the plates of a capacitor, and the depletion layer is like the dielectric, When a diode is reverse-biased, the width of the depletion layer increases with the reverse voltage. Since the depletion layer gets wider with more reverse voltage, the capacitance becomes smaller. It's as though you moved apart the plates of a capacitor. The key idea is that capacitance is controlled by voltage.

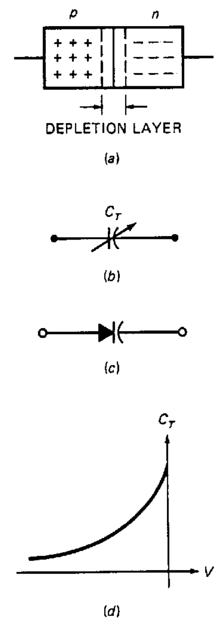


Figure 5–14 — Varactor (a) Structure: (b) Equivalent Circuit; (c) Schematic Symbol; (d) Graph

Figure 5-14*b* shows the equivalent circuit for a reverse-biased diode. At higher frequencies, the varactor acts the same as a variable capacitance. Figure 5-14*d* shows how the capacitance varies with reverse voltage. This graph shows that the capacitance gets smaller when the reverse voltage gets larger. The really important idea here is that reverse voltage controls capacitance. This opens the door to remote control.

Figure 5-14*c* shows the schematic symbol for a varactor. How is this device used? You can connect a varactor in parallel with an inductor to get a resonant circuit. Then you can change the reverse voltage to change the resonant frequency. This is the principle behind tuning in a radio station, a TV channel, etc.

Lightning, power-line faults, etc., can pollute the line voltage by super imposing dips, spikes, and other transients on the normal 115 V rms. *Dips* are severe voltage drops lasting microseconds or less. Spikes are short over voltages of 500 to more than 2000 V. In some equipment, filters are used between the power line and the primary of the transformer to eliminate the problems caused by line transients.

One of the devices used for line filtering is the *varistor* (also called a *transient suppressor*). This semiconductor device is like two back-to-back zener diodes with a high breakdown voltage in both directions. For instance, a V130LA2 is a varistor with a breakdown voltage of 184 V (equivalent to 130 V rms) and a peak current rating of 400 A. Connect one of these across the primary winding, and you don't have to worry about spikes. The varistor will clip all spikes at the 184-V level and protect your equipment.

The Appendix shows the data sheet for the 1N746 series of zener diodes. This data sheet also covers the 1N957 series and the 1N4370 series. Refer to these data sheets during the following discussion. Again, most of the information on a data sheet is for designers, but there are a few items that even troubleshooters and testers will want to know about.

Maximum Power

The power dissipation of a zener diode equals the product of its voltage and current:

$$P_Z = V_Z I_Z \qquad (5\text{-}9)$$

For instance, if V_Z = 12 V and I_Z = 10 mA, then

$$P_Z = (12\text{ V})(10\text{ mA}) = 120\text{ mW}$$

As long as P_Z is less than the power rating, the zener diode can operate in the breakdown region without being destroyed. Commercially available zener diodes have power ratings from 4 to more than 50 W.

For example, the data sheet for the 1N746 series lists a maximum power rating of 400 mW. A safe design includes a safety factor to keep the power dissipation well below this 400-mW maximum. As mentioned elsewhere, safety factors of 2 or more are used for conservative designs.

Maximum Current

Data sheets usually include the maximum current a zener diode can handle without exceeding its power rating. This maximum current is related to the power rating as follows:

$$IZM = \frac{P_{ZM}}{V_Z} \qquad (5\text{-}10)$$

where I_{ZM} = maximum rated zener current

P_{ZM} = power rating

V_Z = zener voltage

For example, the 1N759 has a tenet voltage of 12 V. Therefore, it has maximum current rating of

$$I_{ZM} = \frac{400\text{mW}}{12V} = 33.3\text{mA}$$

The data sheet gives two maximum current ratings: 30 and 35 mA. Notice these values bracket our theoretical answer of 33.3 mA. The data sheet gives you two values because of the tolerance in the tenet voltage.

If you satisfy the current rating, you automatically satisfy the power rating. For instance, if you keep the maximum zener current less than 33.3 mA, you are also keeping the maximum power dissipation less than 400 mW. If you throw in the safety factor of 2, you don't have to worry about a marginal design blowing the diode.

Tolerance

Note 1 on the data sheet shows these tolerances:

lN4370 series: ± 10 percent, suffix A for + 5 percent units

1N746 series: ± 10 percent, suffix A for +5 percent units

1N957 series: ± 20 percent, suffix A for ~ 10 percent units, suffix B for ± 5 percent units

For instance, a 1N758 has a zener voltage of 10 V with a tolerance of ± 10 percent, while the 1N758A has the same zener voltage with a tolerance of +5 percent. The 1N967 has a zener voltage of 18V with a tolerance of ± 20 percent. The 1N967A has the same zener voltages with a tolerance of ± 10 percent, and the 1N967B has the same voltage with a tolerance of ± 5 percent.

Zener Resistance

The tenet resistance (also called zener impedance) may be designated R_{ZT} or Z_{ZT}. For instance, the 1N961 has a tenet resistance of 8.5 Ω. measured at a test current of 12.5 mA. As long as the zener current is above the knee of the curve, you can use 8.5 Ω as the approximate value of the zener resistance. But note how the zener resistance increases at the knee of the curve (700 Ω). The point is this: Operation should be at or near the test current, if at all possible. Then you know the zener resistance is relatively small.

The data sheet contains a lot of additional information, but it is primarily aimed at designers. If you do get involved in design work, then you have to read the data sheet carefully, including the notes that specify how quantities were measured. Data sheets vary from one manufacturer to the next, so you have read between the lines if you want to get to the truth.

Derating

The derating factor shown on a data sheet tells you how much you to reduce the power rating of a device. For instance, the 1N746 series has a power rating of 400 mW for a lead temperature of 50¡C. The derating factor is given as 3.2 mW/ ¡C. This means that you have to subtract 3.2 mW for each degree above 50¡C . Even though you may not be involved in design, you have to be aware of the effect of temperature. If it is known that the lead temperature will be above 50¡C, the designer has to derate or reduce the power rating of the zener diode.

Troubleshooting

Figure 5-15 shows a zener regulator. When the circuit is working properly, the voltage between node A and ground is +18 V, the voltage between A node B and ground is + l0 V, and the voltage between node C and ground is + 10 V.

Now, let's discuss what can go wrong with the circuit. When a circuit is not working as it should, a troubleshooter usually starts by measuring node voltages. These voltage measurements give clues that help isolate the trouble. For instance, suppose he or she measures these node voltages

$$V_A = +18\text{ V} \quad V_B = +10\text{ V} \quad V_C = 0$$

When you are trying to figure out what causes incorrect voltages, trial and error is useful. That is, you play the what-if game. Here is what may go through a troubleshooter's mind after measuring the foregoing node voltages.

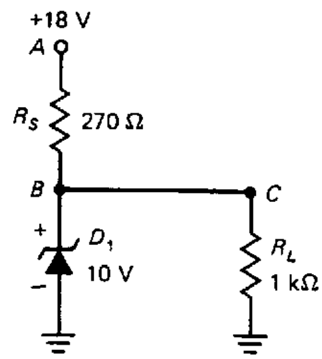


Figure 5–15
Zener Regulator

What if the load resistor were open? No, the load voltage would still be + 10 V. What if the load resistor were shorted? No, that would pull nodes B and C down to ground, producing 0 V. All right, what if the connecting wire between nodes B and C were open? Yes, that would do it. That's got to be it.

This trouble produces unique symptoms. The only way you can get this set of voltages is with an open connection between nodes B and C.

Not all troubles produce unique symptoms. Sometimes, two or more troubles produce the same set of voltages. Here is an example. Suppose the troubleshooter measures these node voltages:

$$VA = +18V\ V_B = 0\ V_C = 0$$

What do you think the trouble is? Think about this for a few minutes. When you have an answer, read what follows.

Here is a way that a troubleshooter might find the trouble. The thinking goes like this:

I've got voltage at A, but not at B and C. What if the series resistor were open? Then no voltage could reach node B or node C, but I would still measure + 18 V between node A and ground. Yes, the series resistor is probably open.

At this point, the troubleshooter would disconnect the series resistor and measure its resistance with an ohmmeter. Chances are that it would be open. But suppose it measures okay. Then the troubleshooter's thinking continues like this:

That's strange. Well, is there any other way I can get +18 V at node A and 0 V at nodes B and C? What if the zener diode were shorted? What if the load resistor were shorted? What if a solder splash were between node B or node C and ground. Any of these will produce the symptoms I'm getting.

Now, the troubleshooter has more possible troubles to check out. Eventually, she or he will find the trouble.

When components burn out, they usually become open, but not always. Some semiconductor devices can develop internal shorts, in which case, they are like zero resistances. Other ways to get shorts include a solder splash between traces on a printed-circuit board, a solder ball touching two traces, etc. Because of this, you must include what-if questions in terms of open components, as well as open components.

Example 5-8

Assume an ideal zener diode and work out the node voltages for all possible shorts and opens in Fig. 5-15.

Solution

In working out the voltages, remember this. A shorted component is equivalent to a resistance of zero, while an open component is equivalent to a resistance of infinity. If you have trouble calculating with 0 and ∞ then use 0.001 Ω and 1000 MΩ . In other words, use a very small resistance for a short and a very large resistance for an open.

To begin, the series resistor R_S may be shorted or open. Let us designate these R_{SS} and R_{SO}, respectively. Similarly, the zener diode may be shorted or open, symbolized by D_{1S} and D_{10}. Also, the load resistor may be shorted or open, R_{LS} and R_{LO}. Finally, the connecting wire between B and C may be open, designated BC_O.

If the series resistor were shorted, + 18 V would appear at nodes B and C. This would destroy the zener diode and possibly the load resistor, but the voltage would remain at + 18 V. Then a troubleshooter would measure V_A = +18 V, V_B = + 18V, and V_C = +18V. This trouble and its voltages are shown in Table 5-1.

If the series resistor were open, then the voltage could not reach node B. In this case, nodes B and C would have zero voltage. Continuing like this, we can get the remaining entries shown in Table 5-1.

In Table 5-1, the comments indicate troubles that might occur as a direct result of the original short circuits. For instance, a shorted R_S will destroy the zener diode and may also burn out the load resistor. It depends on the power rating of the load resistor. A shorted R_S means there's 18 V across kΩ. This produces a power of 0.324 W. If the load resistor is rated at only 0.25 W, it will burn out.

Study the table. You can learn a lot from it. Also, use the T-shooter at the end of this chapter to practice troubleshooting a zener regulator.

Trouble	V_A, V	V_B, V	V_C, V	Comments
None	18	10	10	No trouble.
R_{SS}	18	18	18	D_1 and R_L may be blown.
R_{SO}	18	0	0	
D_{1S}	18	0	0	R_S may be blown.
D_{1O}	18	14.2	14.2	
R_{LS}	18	0	0	R_S may be blown.
R_{LO}	18	10	10	
BC_O	18	10	0	
No supply	0	0	0	Check power supply.

Table 5–1
Zener Regulator Troubles and Symptoms

Optional Topics

The following material continues the earlier discussions at a more advanced and specialized level. All the topics are optional because they are not used in any of the basic discussions in later chapters. This section will be a useful reference when you are in industry because then you will probably want more advanced viewpoints.

Load Lines

The current through the zener diode of Fig. 5-16*a* is given by

$$I_S = \frac{V_S - V_Z}{R_S} \qquad (5\text{-}11)$$

This says the zener current equals the voltage across the series resistor divided by the resistance. Equation (5-11) can be used to construct load line as previously discussed. For instance, suppose V_S = 20 V and R_S = 1 k Ω. Then the foregoing equation reduces to

$$I_S = \frac{20 - V_Z}{1000}$$

As before, we get the saturation point (vertical intercept) by setting V_Z equal to zero and solving for I_Z to get 20 mA. Similarly, to get the cutoff point (horizontal intercept), we set I_Z equal to zero and solve for V_Z to get 20 V.

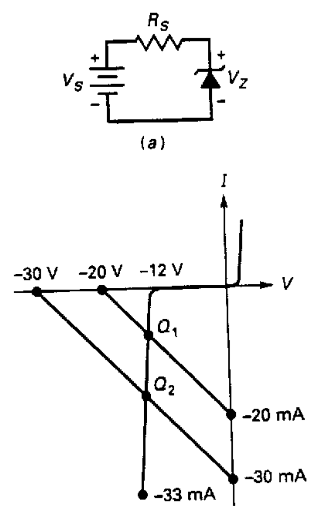


Figure 5–16 Zener Diode Circuit

The following study aids will help to reinforce the ideas discussed in this chapter. For best results, use these study aids within 6 hours of reading the earlier material. Then review these study aids a week later and month later to ensure that the concepts remain in your long-term memory.

Summary

Sec. 5-1 The Zener Diode

This is special diode optimized for operation in the breakdown region. Its main use is in voltage regulators, circuits that hold the load voltage constant. Ideally, a zener diode is like a perfect battery. To a second approximation, it has bulk resistance that produces a small additional voltage.

Sec. 5-2 The Loaded Zener Regulator

When a zener diode is in parallel with a load resistor, the current through the current-limiting resistor equals the sum of the zener current and the load current. The process for analyzing zener regulator consists of finding the series current, load current, and zener current (in that order.)

Sec. 5-3 Optoelectronic Devices

The LED is widely used as an indicator on instruments, calculators, and other electronic equipment. By combining seven LEDs in a package, we get a seen-segment indicator. Another important optoelectronic device is the optocoupler, which allows us to couple a signal between two isolated circuits.

Sec. 5-4 The Schottky Diode

The reverse recovery time is the time it takes a diode to shut off after it is suddenly switched from forward to reverse bias. This time may only be a few nanoseconds, but it places a limit on how high the frequency can be in rectifier circuit. The Schottky diode is a special diode with almost zero reverse recovery time. Because of this, the Schottky diode is useful at high frequencies where short switching times are needed.

Sec. 5- 5 The Veractor

The width of the depletion layer increases with the reverse voltage. This is why the capacitance of a varactor can be controlled by the reverse voltage. This leads to remote tuning of radio and television sets.

Sec. 5-6 Varistors

These protective devices are used across the primary winding of a transformer to prevent voltage spikes from damaging or otherwise polluting the in and out voltage to the equipment.

Sec. 5-7 Reading a Data Sheet

The most important quantities on the data sheet of zener diodes are the zener voltage, the maximum power rating, the maximum current rating, and the tolerance. Designers also need the zener resistance, the derating factor, and a few other items.

Sec. 5-8 Troubleshooting

Troubleshooting is an art and a science. Because of his, you can only learn so much from a book. The rest has to be learned from direct experience with circuits in trouble. Because trouble-shooting is an art, you have to ask What if? Often and feel your way to a solution.

Vocabulary

In your own words, explain what each of the following terms mean. Keep your answers short and to the point. If necessary, verify your answer by rereading the appropriate discussion or by looking at end-of-book Glossary.

Light emitting diode (LED)	temperature coefficient
open	varactor
optocoupler	varistor
photodiode	voltage regulator
photodiode	zener resistance
process	zener voltage
Schottky diode	short

The following formulas are useless if you don't know what they mean in words. Suggestion: Look at each formula, then read the words to find out what it means.. Your chances of learning and remembering are much better if you concentrate on words rather than formulas.

Eq. 5-1 Current through Series Resistor

$$I_S = \frac{V_S - V_Z}{R_S}$$

This is an equation that you do not have to memorize. It says the current through the series resistor equals the voltage across the series resistor divided by the resistance. It is another example if Ohm's law, where the voltage is the difference of the node voltages of the ends of a resistor.

Eq. 5-2 Thevenin Voltage

$$V_{TH} = \frac{R_L}{R_S + R_L} V_S$$

This is the voltage across the load resistor when the zener diode is disconnected. One way to remember it this: V_S divided by $R_S + R_L$ is the load current. Multiply this load current by R_L and you get V_{TH}. The value of V_{TH} has to be larger than the zener voltage to get voltage regulation.

Eq. 5-6 Zenner Current

$$I_Z = I_S - I_L$$

This is disguised form of Kirchhoff's current law. It says the zener current equals the difference between the series current and load current. To use it, you must already have carried out the two preceding steps in the process: 1. Find I_{S2}. Find I_L.

Eq. 5-7 Zenner Power

$$P_Z = V_Z I_Z$$

The zener power equals the zener voltage times the zener current. This power has to be less than the maximum power rating listed on the data sheet. Otherwise, you may burn out or seriously degrade the characteristics of the zener diode.

Eq. 5-8 LED Current

$$IS = \frac{V_S - V_D}{R_S}$$

This gives you the current through a resistor in series with a LED. It says the current equals the voltage across the series resistor divided by the resistance. Use 2 V for the value of V_D, unless you have a more accurate value for the voltage across the LED.

The following may have more than one right answer, Select the best answer. This is the one that is always true, or covers more situations, etc.

1. What is true about the breakdown voltage in a zener diode?
    a. It decreases when current increases.
    b. It destroys the diode.
    c. It equals the current times the resistance.
    d. It is approximately constant.
2. Which of these is the best description of a zener diode?
    a. It is a diode.
    b. It is a constant-voltage device.
    c. It is a constant-current device.
    d. It works in the forward region.
3. A zener diode
    a. Is a battery
    b. Acts like a battery in the breakdown region
    c. Has a barrier potential of 1 V
    d. Is forward-biased
4. The voltage across the zener resistance is usually
    a. Small
    b. Large
    c. Measured in volts
    d. Subtracted from the breakdown voltage
5. If the series resistance decreases in an unloaded zener regulator, the zener current
    a. Decreases
    b. Stays the same
    c. Increases
    d. Equals the voltage divided by the resistance

6. In the second approximation, the total voltage across the zener diode is the sum of the breakdown voltage and the voltage across the

    a. Source

    b. Series resistor

    c. Zener resistance

    d, Zenerdiode

7. The load voltage is approximately constant when a zener diode is

    a. Forward-biased

    b. Reverse-biased

    c. Operating in the breakdown region

    d. Unbiased

8. In a loaded zener regulator, which is the largest current?

    a. Series current

    b. Zener current

    c. Load current

    d. None of these

9. If the load resistance decreases in a zener regulator, the zener current

    a. Decreases

    b. Stays the same

    c. Increases

    d. Equals the source voltage divided by the series resistance

10. If the load resistance decreases in a zener regulator, the series current

    a. Decreases

    b. Stays the same

    c. Increases

d. Equals the source voltage divided by the series resistance

11. When the source voltage increases in a zener regulator, which of these currents remains approximately constant?

    a. Series current

    b. Zener current

    c. Load current

    d. Total current

12. If the zener diode in a zener regulator is connected with the wrong polarity, the load voltage will be closest to

    a. 0.7 V

    b. 10 V

    c. 14 V

    d. 18 V

13. At high frequencies, ordinary diodes don't work properly because of

    a. Forward bias

    b. Reverse bias

    c. Breakdown

    d. Charge storage

14. The capacitance of a varactor diode increases when the reverse voltage across it

    a. Decreases

    b. Increases

    c. Breaks down

    d. Stores charges

15. Breakdown does not destroy a zener diode, provided the zener current is less than the

    a. Breakdown voltage

    b. Zener test current

    c. Maximum zener current racing

    d. Barrier potential

16. To display the digit 8 in a seven-segment indicator,

    a. *C* must be lighted

    b. *G* must be off

    c. *F* must be on

    d. All segments must be lighted

17. A photo diode is normally

    a. Forward-biased

    b. Reverse-biased

    c. Neither forward- nor reverse-biased

    d. Emitting light

18. When the light increases, the reverse minority-carrier current in a photodiode

    a. Decreases

    b. Increases

    c. Is unaffected

    d. Reverses direction

19. The device associated with voltage-controlled capacitance is a

    a. LED

    b. Photodiode

    c. Varactor diode

    d. Zenerdiode

20. If the depletion layer gets wider, the capacitance

    a. Decreases

    b. Stays the same

    c. Increases

    d. Is variable

21. When the reverse voltage increases, the capacitance

    a. Decreases

    b. Stays the same

    c. Increases

    d. Has more band width

22. The varactor is usually

    a. Forward-biased

    b. Reverse-biased

    c. Unbiased

    d. In the breakdown region

Basic Problems

Sec. 5-1 The Zener Diode

5-1. An unloaded zener regulator has a source voltage of 20V, a series resistance of 330 Ω, and a zener voltage of 12 V. What is the zener current?

5-2. If the source voltage in Prob. 5-1 varies from 20 to 40 V, what is the maximum zener current?

5-3. If the series resistor of Prob. 5-1 has a tolerance of $\pm$10 percent, what is the maximum zener current?

Sec. 5-2 The Loaded Zener Regulator

5-4. If the zener diode is disconnected in Fig. 5-23, what is the load voltage?

5-5. Assume the supply voltage of Fig. 5-23 decreases from 20 to O V. At some point along the way, the zener diode will stop regulating: Find the supply voltage where regulation is lost.

5-6. Calculate all three currents in Fig. 5-23.

5-7. Assuming a tolerance of $\pm$ 10 percent in both resistors of Fig. 5-23, what is the maximum zener current?

5-8. Suppose the supply voltage of Fig. 5-23 can vary from 20 to 40 V. What is the maximum zener current?

5-9. What is the power dissipation in the resistors and zener diode of Fig. 5-23?

5-10. The zener diode of Fig. 5-23 is replaced with a IN961. What are the load voltage and the zener current?

5-11. The zener diode of Fig. 5-23 has a zener resistance of 11.5 Ω. If the power supply has a ripple of I V, what is the ripple across the load resistor?

5-12. Draw the schematic diagram of a zener regulator with a supply voltage of 25 V, a series resistance of 470 Ω, a zener voltage of I5 V, and a load resistance of 1 kΩ. What are the load voltage and the zener current?

Sec. 5-3 Optoelectronic Devices

5-13. What is the current through the LED of Fig. 5-24?

5-14. If the supply voltage of Fig. 5-24 increases to 40 V, what is the LED current?

5-15. If the resistor is decreased to 1 kΩ, what is the LED current in Fig. 5-24?

5-1. 24.2 mA

5-3. 26.9 mA

5-5. 14.6 V

5-7. 19.6 mA

5-9. Ps is 194 mW, PL is 96 mW, and Pt is 195 mW

5-11. 33.7 mV

5-13. 5.91 mA

5-15. 13 mA

5-17. 200 mW

5-19. 11.4 V, 12.6 V

5-21. a. O b. 16.4 V c. O d. O

5-23. Check for a short across the 330 R.

5-25. 12.2 V

5-27. Many designs are possible here. One design is a 1N754, a series resistance of 270 R, and a load resistance of 220 R. This design results in a series current of 48.9 mA, a load current of 30.9 mA, zener current of 18 mA.

5-29. 26 mA

5-31. 7.98 V

5-33. Trouble 2: Wire ED open

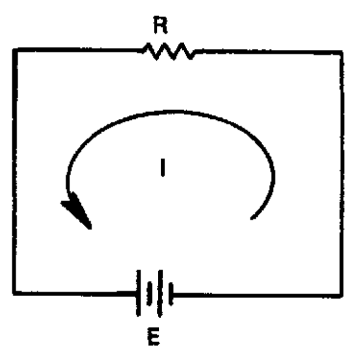
5-35. Trouble 5: No supply voltage

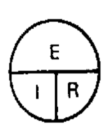
Ohm's Law and Power

The following examples are designed to reinforce your understanding of the use of Ohm's law and power formulas with scientific notation.

All of the examples involve the use of powers of ten with one exception. Example 5 illustrates the proper use of the P-1^2-R circle formula to find the current flowing in a simple circuit. This involves taking the square root of a number. Since the procedure for finding square roots of quantities expressed in scientific notation is covered in Lesson 10, only very simple numbers are used in the example.

1. In the simple circuit shown, a voltage is applied to a resistor and current flows through the resistor. Use Ohm's law to find the applied voltage if the current is 5 mA and the resistance is 3 kilohms.





a. Draw the circle formula for Ohm's law.

b. Cover the quantity you want to find with your thumb; in this case, cover E. Remember, a vertical line tells you to multiply the quantities on either side of the line and a horizontal line tells you to divide the bottom quantity into the top.

$E = 1 \times R$

c. The resulting formula is E = 1 X R.

$E = 5 \text{ mA} \times 3\text{k}\Omega$

d. Substitute the values of voltage and current in the formula

$E = 5 \times 10^{-3} \times 3 \times 10^{+3}$

e. Convert to powers of ten.

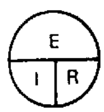
$E = 15 \times 10^{0}$

f. Multiply the leading numbers and combine the exponents. In this case, +3 and -3 equal zero.

$E = 15 \text{ volts}$

g. Convert to metric prefixed form. Because 10^{0} = 1, the answer can best be expressed directly in units, E = 15 volts.

2. Given the same circuit as in Example 1, use Ohm's law to find the current flowing when the voltage is 12.6 volts and the resistance is 820 ohms.



a. Draw the circle formula for Ohm's law.

b. Cover the quantity you want to find with your thumb; in this case, cover I.

$I = \frac{E}{R}$

c. The resulting formula is $I = \frac{E}{R}$

$I = \frac{12.6 \ V}{820}$

d. Substitute the values of voltage and resistance in the formula.

$I = \frac{1.26 \ X \ 10^{+1}}{8.2 \ X \ 10^{+2}}$

e. Convert to powers of ten.

$I = \frac{1.26 \ X \ 10^{+1-2}}{8.2}$

f. Bring the bottom exponent across the division line, up to the top and change its sign.

$I = \frac{1.26 \ X \ 10^{-1}}{8.2}$

g. Combine the exponents. Here a +1 and a -2 equal -1.

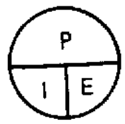
$I = .154 \times 10^{-1}$

h. Divide the leading numbers and round off.

$I = 15.4 \text{ mA}$

I. Convert to metric prefixed form.

3. Again considering the same circuit as before, find the power dissipated by the resistor when the applied voltage is 45 volts and the current flowing through the resistor is 16 mA.



a. Draw the P-I-E circle formula for power.

b. Cover the quantity you want to find with your thumb; in this case, cover P.

$P = I \times E$

c. The resulting formula is P = I X E.

$P = 16\ mA \times 45\ V$

d. Substitute the values of voltage and current in the formula.

$P = 16 \times 10^{-3} \times 4.5 \times 10^{+1}$

e. Convert to powers of ten.

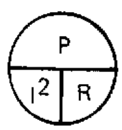
$P = 72 \times 10^{-2}$

f. Multiply the leading numbers and combine the exponents. In this case, -3 and +1 equal -2.

$P = 720\ mW$

g. Convert to metric prefixed form. A 1-watt resistor would be appropriate for this example.

4. In a simple circuit, find the power dissipated by a 100-ohm resistor when the current flowing through it is 50 mA.



a. Since you *know* I and R, and *need to find* P, select the P-1^2-r circle formula for power.

b. Cover the quantity you want to find with your thumb; in this case, cover P.

$P = 1^2 \times R$

c. The resulting formula is $P = 1^2 \times R$.

$P = (50\ MA)^2 \times 100\Omega$

d. Substitute the values of current and resistance in the formula.

$P = (5 \times 10^{-2}) \times (5 \times 10^{-2}) \times 1 \times 10^{+2}$

e. Convert to powers of ten. Since 50 mA equal 5×10^{-2}, the square of 50 mA equals 5×10^{-2} times 5×10^{-2}.

$P = 25 \times 10^{-4} \times 1 \times 10^{+2}$

f. Multiply 5×10^{-2} by itself remembering to add the exponents. Here -2 and -2 equals -4.

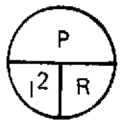
$P = 25 \times 10^{-2}$

g. Multiply again and add the exponents. Here -4 and +2 equals -2.

$P = 250\ mW$

h. Convert to metric prefixed form. A 1/2-watt resistor would be used in this example.

5. Given a simple circuit, find the current flowing through a 4-ohm resistor when the resistor is dissipating 100 watts of power.



a. Here you know P and R, and need 1, so draw the P-1^2-R circle formula.

b. Cover the quantity you want to find with your thumb; in this case cover I^2.

$I^2 = \dfrac{P}{R}$

c. The resulting formula is $I^2 = \dfrac{P}{R}$.

$I^2 = \dfrac{100\ W}{4\ \Omega}$

d. Substitute the values of power and resistance in the formula.

$I^2 = 25$

e. Divide; note that the result is the square of current.

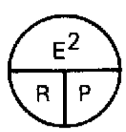
$I = \sqrt{25}$

f. To get the current, you must then find the square root of 25.

$I = 5$ A

g. If you have a calculator with a square root key, enter 25, press the square root key, and the answer, 5, will appear in the display. You may also use the square root tables in the Appendix. To find the square root of 25, look up 25 in the table, and look across to the column labeled square roots ($\sqrt{}$) where you should see "5."

6. If the voltage applied to a 3.3 kilohm resistor in a simple circuit is 15 volts, find the power dissipated by the resistor.



a. Here you know E and R and need to find P, so draw the E^2-R-P circle formula.

b. Cover the quantity you want to find with your thumb; in this case, cover P.

$P = \dfrac{E^2}{R}$

c. The resulting formula is $P = \dfrac{E^2}{R}$.

$P = \dfrac{(15\ V)^2}{3.3\ k\Omega}$

d. Substitute the values of voltage and resistance in the formula.

$P = \dfrac{(1.5\ X\ 10^{+1})\ X\ (1.5\ X\ 10^{+1})}{3.3\ X\ 10^{+3}}$

e. Convert to powers of ten. Since 15 volts equals 1.5 X 10^{+1}, the square of 15 volts equals 1.5 X 10^{+1} times 1.5 X 10^{+1}.

$P = \dfrac{2.25\ X\ 10^{+2}}{3.3\ X\ 10^{+3}}$

f. Square 1.5 X 10^{+1} by multiplying it by itself, remembering to add the exponents.

P = $\dfrac{2.25\ X\ 10^{+2-3}}{3.3}$	g. Bring the bottom exponent across the division line, up to the top and change its sign.
P = $\dfrac{2.25\ X\ 10^{-1}}{3.3}$	h. Combine the exponents; here +2 and -3 equals -1.
P = .682 X 10^{-1}	I. Divide the leading numbers and round off.
P = 68.2 mW	j. Convert to metric prefixed form.

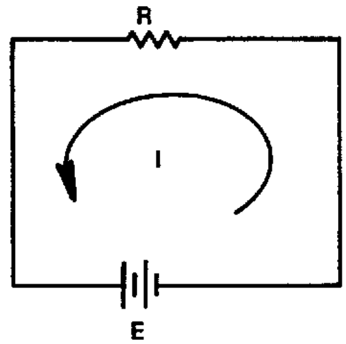
7. In the simple circuit shown below, the applied voltage forces current to flow through the resistor. If the voltage is increased while the resistance remains constant, the current will increase. Remember, in a circuit with a constant resistance, voltage and current vary directly.
   On the chart next to the circuit, the increase in voltage is indicated by an arrow pointing up (↑), the constant resistance is indicated by a dot (•), and the resulting increase in current flow is also indicated by an arrow point up (↑). In a *direct* relationship, when one quantity increases, the other quantity decreases. Using this information and considering the simple circuit shown, complete the chart by filling in the blank spaces with the appropriate symbol:

   ↑ means the quantity increases

   ↓ means the quantity decreases

   • means the quantity remains constant

	E	I	R
Example	↑	↑	●
1	●	↑	
2	↓		●
3	●	↓	
4	●		↓
5		●	↑

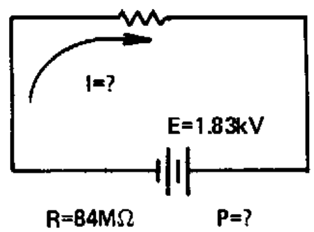


n:

Solution	E	I	R
1	●	↑	↓
2	↓	↓	●
3	●	↓	↑
4	●	↑	↓
5	↑	●	↑

Practice Problems

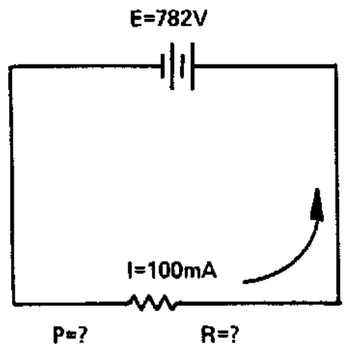
1.



I = ____________

P = ____________

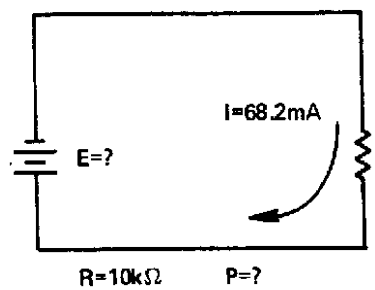
2.



P = ____________

R = ____________

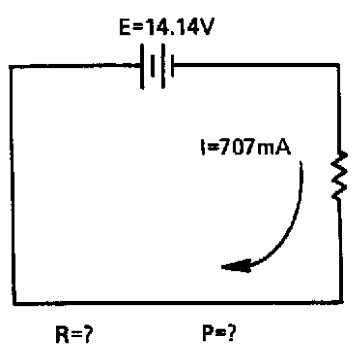
3.



E = ____________

P = ____________

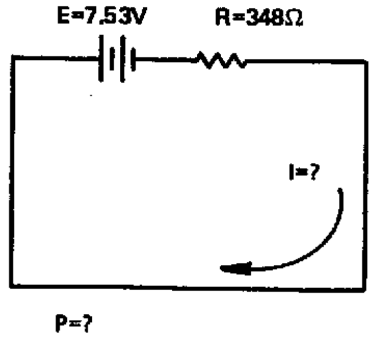
4.



R = ____________

P = ____________

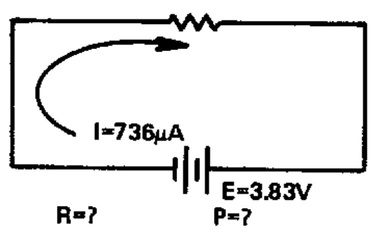
5.



I = ____________

P = ____________

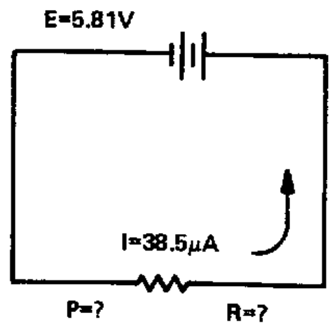
6.



R = ____________

P = ____________

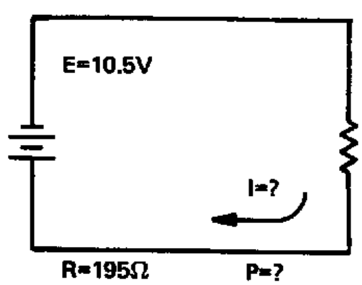
7.



P = ____________

R = ____________

8.



I = ____________

P = ____________

Solutions for Odd Numbered Questions

1. $I = 21.8\ \mu A$

   $P = 39.9\ mW$

2. $P = 78.2\ W$

   $R = 7.82\ k\Omega$

3. $E = 682\ V$

   $P = 46.5\ W$

4. $R = 20.0\ \Omega$

   $P = 10\ W$

5. $I = 21.6\ mA$

   $P = 163\ mW$

6. $R = 5.2\ k\Omega$

   $P = 2.82\ mW$

7. $P = 224\ W$

   $R = 151\ k\Omega$

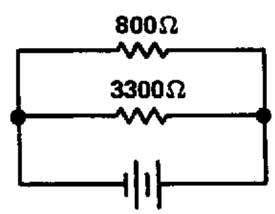
8. $I = 53.8\ mA$

   $P = 565\ mW$

Introduction to Parallel Circuits

Worked Through Examples

1. Find the total resistance of the following circuit.



There are two options that may be taken to find R_T. The product-over-sum formula or the sum of the reciprocal formula. This first example will use the product-over-sum formula:

$$R_T = \frac{R_1 \; X \; R_2}{R_1 \; + \; R_2}$$

First, substitute the circuit values in correct powers of ten form.

$$R_T = \frac{8.0 \; X \; 10^2 \; X \; 3.3 \; X \; 10^3}{8.0 \; X \; 10^2 \; + \; 33.0 \; X \; 10^2}$$

To *add,* the exponents of the numbers in the denominator (or bottom) of this equation must be the same. Changing the 3.3 X 10^{+3} to 33.0 X 10^{+2}, you have

$$R_T = \frac{8.0 \; X \; 10^2 \; X \; 3.3 \; X \; 10^3}{8.0 \; X \; 10^2 \; + \; 33.0 \; X \; 10^2}$$

Add

$$R_T = \frac{8.0 \ X \ 10^2 \ \ X \ 3.3 \ X \ 10^3}{41 \ + \ 10^2}$$

Multiply the numbers on top. (Remember to *add* the exponents when multiplying.)

$$R_T = \frac{26.4 \ X \ 10^5}{41 \ X \ 10^2}$$

Now you may divide 26.4 $X10^5$ by 41 X 10^2. (Remember to do this you bring the bottom exponent up above the division line and change its sign.)

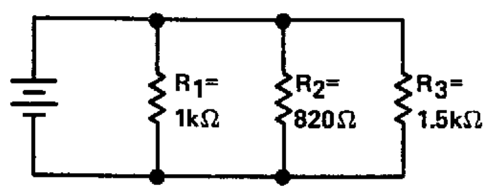
$$R_T = \frac{26.4 \ X \ 10^{5-2}}{41}$$

Then combine these top exponents

$$R_T = \frac{26.4 \ X \ 10^3}{41}$$

$$RT = 6.44 \text{ X } 10^2 = 644 \ \Omega$$

2. Find the total resistance of the circuit shown below.



This time the reciprocal formula will be used to solve this problem.

First, substitute the circuit values into the formula:

$$R_T = \frac{1}{1/R_1 \ + \ 1/R_2 \ + \ 1/R_3}$$

$$R_T = \frac{1}{\frac{1}{1.0 \ X \ 10^3} + \frac{1}{8.2 \ X \ 10^2} + \frac{1}{1.5 \ X \ 10^3}}$$

Find the *reciprocals* of the resistance values. (Divide the resistance value into 1.) This gives you the individual conductances which go into the bottom of this equation.

$$R_T = \frac{1}{1\ X\ 10^{-3} + 1.22\ X\ 10^{-3} + 6.67\ X\ 10^{-4}}$$

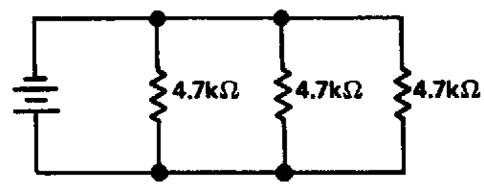
Add all individual conductances in the bottom of this equation. (Remember to change all exponents to the same number; here, 10^{-3}.)

$$R_T = \frac{1}{2.89\ X\ 10^{-3}}$$

Now divide 2.89 X 10^{-3} into 1 to find the total resistance.

$$R_T = 3.46 \text{ X } 10^2 = 346\ \Omega$$

3. Find the approximate resistance of the circuit shown below. (Use the quickest method.)



Since the three resistors are equally sized, the "shortcut" formula may be used.

$$R_{eq} = \frac{R_5}{N}$$

R^5 = Same size resistor resistance (4.7 kΩ)

N = Number of resistors (3).

Substituting

$$R_{eq} = \frac{4.7\ k\Omega}{3}$$

Change 4.7 kΩ to proper powers of ten notation

$$R_{eq} = \frac{4.7\ X\ 10^3}{3}$$

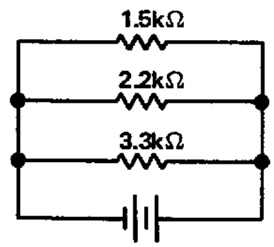
Divide

$$R_{eq} = 1.57 \text{ X } 10^3 = 1570\ \Omega$$

4. Define the term "Branch."

A branch in an electrical circuit is simply a separate path through which electrical current can flow. In other words, a series circuit has only one branch. A parallel circuit has two or more branches.

5. Find the R_{eq} of the following circuit.



This problem will be worked using an SR-50 type calculator. This reciprocal formula will be used to solve the problem.

Enter the first number in correct powers of ten form.

1.5 $\boxed{\text{EE}}$ 3

Press the reciprocal key and store that number in the calculator's memory.

$\boxed{1/X}$ $\boxed{\text{STO}}$

Enter the other two numbers using the same procedure as outlined above except rather than pressing the "STO" key, press the Σ key which adds the displayed number to the number held in memory.

2.2 $\boxed{\text{EE}}$ 3 $\boxed{1/X}\boxed{\Sigma}$

3.3 $\boxed{\text{EE}}$ 3 $\boxed{1/X}\boxed{\Sigma}$

The reciprocals of all three numbers have been found and added together.

This number may be recalled by pressing the "RCL" key.

$\boxed{\text{RCL}}$

Now, this number must be divided into 1, so press the reciprocal key.

$\boxed{1/X}$

Your answer appears on the display.

7.021276596 02

This number is rounded to

7.02 X 102 or

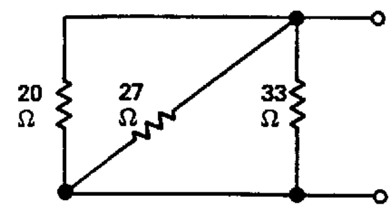
702 Ω

The key objective of this lesson has been achieved if you can calculate the total resistance of any basic parallel circuit. To gain some practice in this area, the problems below are provided.

Depending upon the approach you use to solve these problems and how you round off intermediate results, your answers may vary slightly from those given here. However, any differences you encounter could only occur in the third significant digit of your answer. If the first two significant digits of your answers do not agree with those given here, recheck your calculations.

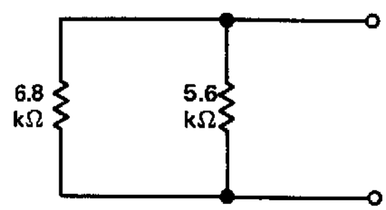
Find RT for each of the following circuits.

1.



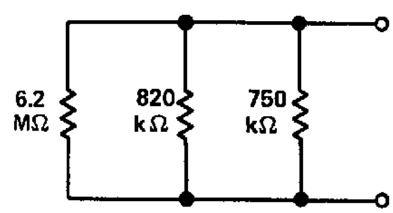
R_T = ________

2.



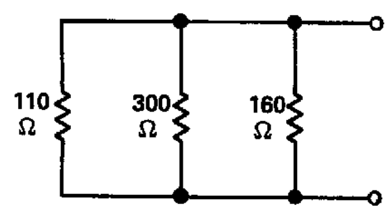
R_T = ________

3.



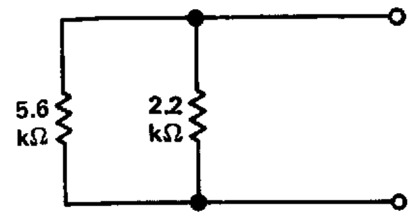
R_T = ________

4.



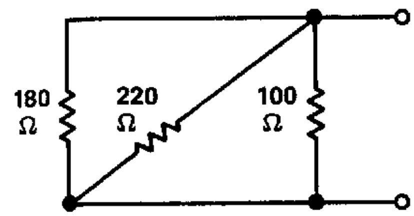
R_T = ________

5.



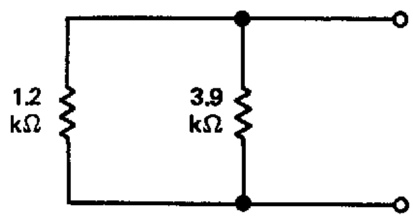
R_T = ________

6.



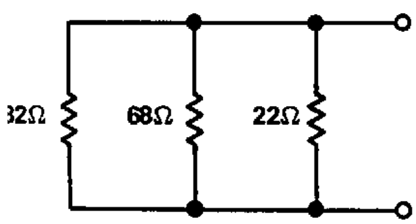
R_T = ________

7.



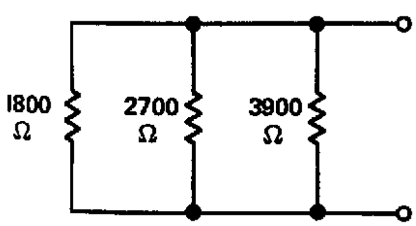
R_T = ________

8.



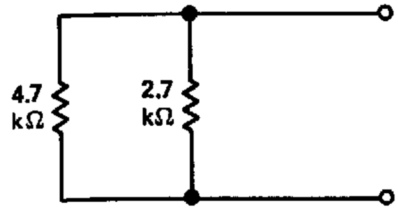
R_T = ________

9.



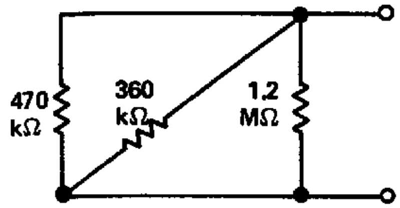
R_T = ________

10.



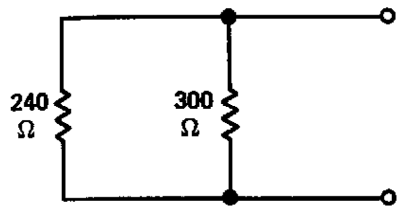
R_T = ________

11.



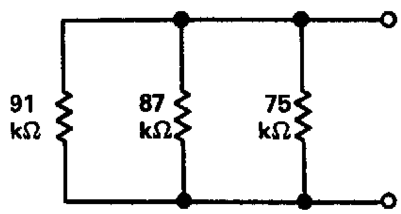
R_T = ________

12.



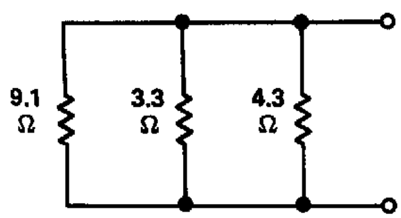
R_T = ________

13.



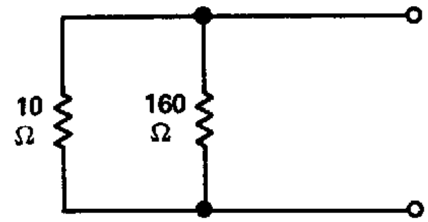
R_T = ________

14.



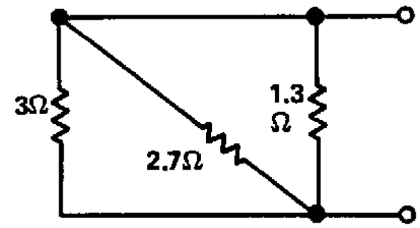
R_T = ________

15.



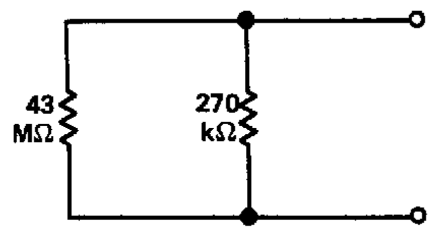
$R_T =$ __________

16.



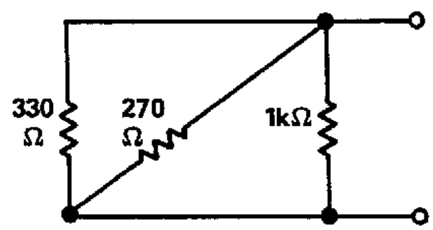
$R_T =$ __________

17.



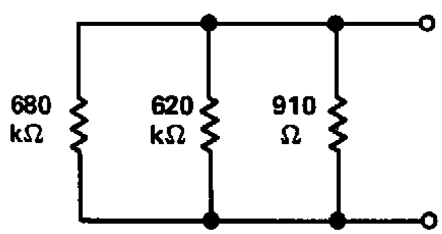
$R_T =$ __________

18.



$R_T =$ __________

19.



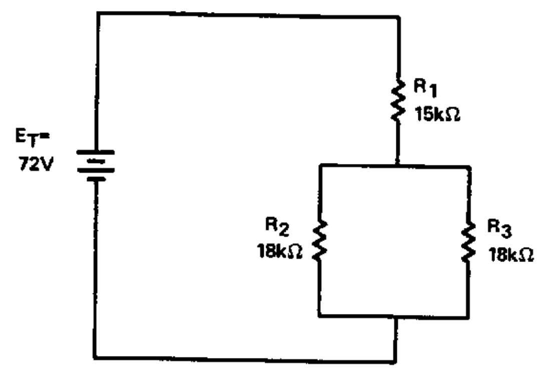
$R_T =$ __________

1. 8.52 Ω
2. 3.07 kΩ
3. 368 kΩ
4. 53.5 Ω
5. 1.58 kΩ
6. 49.7 Ω
7. 918 Ω
8. 13.8 Ω
9. 846 Ω
10. 1.71 kΩ
11. 174 kΩ
12. 133 Ω
13. 27.9 kΩ
14. 1.55 Ω
15. 9.41 Ω
16. 679 mΩ
17. 268 kΩ
18. 129 Ω
19. 907 Ω

Series-Parallel Circuits

Worked Through Examples

1. In the series-parallel circuit shown, calculate the total equivalent resistance and all unknown voltages and currents using Ohm's law and circuit reduction techniques.



First, you can find R_T by circuit reduction techniques. Since R_2 and R_3 are of equal value and are connected in parallel, the equivalent resistance, $R_{2,3}$ can be found with the formula:

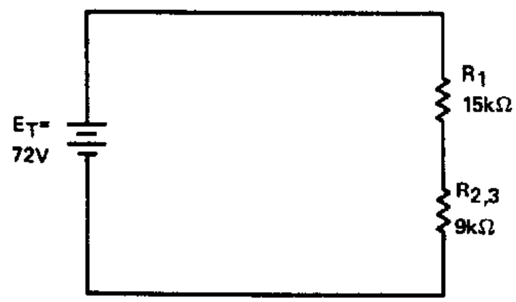
$$R_{eq} = \frac{R_S}{N}$$

R_S equals 18 kilohms and N equals 2, so:

$$R_{2,3} = R_{eq} = \frac{R_S}{N} = \frac{18\ k\Omega}{2}$$

$$R_{2,3} = 9\ \text{k}\Omega$$

After the first circuit reduction, the circuit now consists of R_1 in series with $R_{2,3}$ as shown.



You can find the total resistance of the circuit by simply using the series circuit law which says that the total resistance of a series circuit equals the sum of the individual resistances. In formula form:

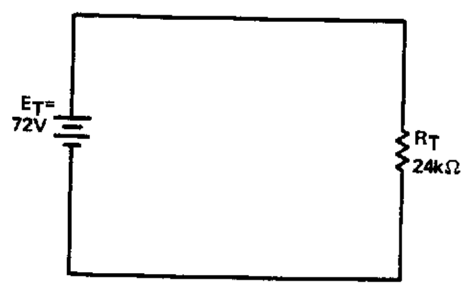
$$R_T = R_1 + R_2 + R_3 + \ldots$$

or in this case:

$$R_T = R_1 + R_{2.3}$$

$$R_T = 15\ k\Omega + 9\ k\Omega$$

$$R_T = 24\ k\Omega$$



Once you know the total resistance, you can find the total current by using Ohm's law in the form $I_T = E_T/R_T$. Substituting the appropriate values in the formula gives:

$$I_T = \frac{E_T}{R_T} + \frac{72\ V}{24\ k\Omega}$$

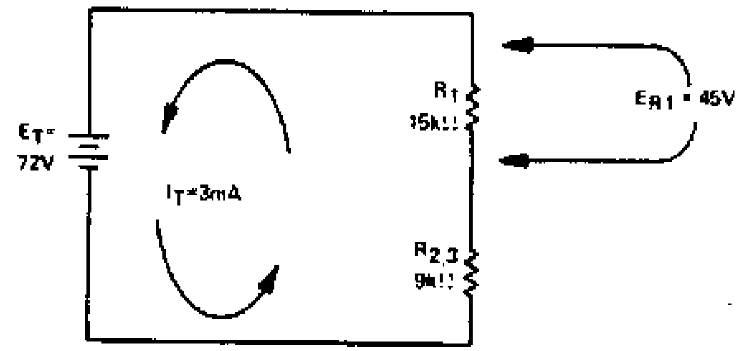
$$I_T = 3\ mA$$

This total current can be used to find the voltage across R_1. Remember, since R_1 is in series with the rest of the circuit, the total current must flow through R_1. If you use Ohm's law in the form E = 1 X R and substitute the appropriate values, you get:

$$E_{R1} = I_T \times R_1$$

$$E_{R1} = 3\text{ mA} \times 15\text{ k}\Omega$$

$$E_{R1} = 45\text{ V}$$



Remember that in a series circuit the total voltage equals the sum of the individual voltage drops. You know the total voltage and the voltage across R_1; the remainder of the voltage must be dropped across $R_{2,3}$. In formula form:

$$E_{R2,3} = E_T - E_{R1}$$

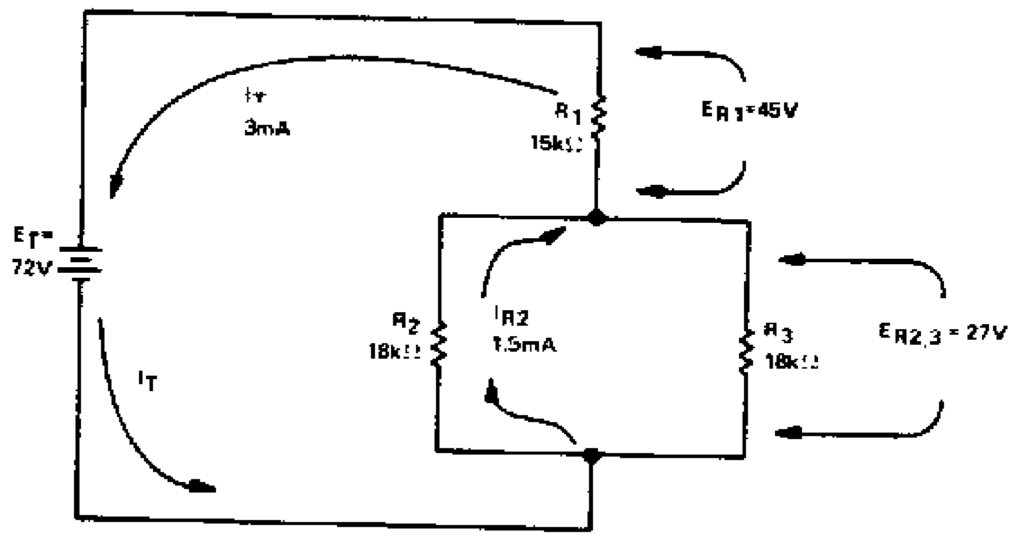
$$E_{R2,3} = 72\text{ V} - 45\text{ V}$$

$$E_{R2,3} = 27\text{ V}$$

You can find the current through R_2 or R_3 by using Ohm's law in the form I = E/R. Remember, R_2 and R_3 are in parallel, so they have the same 27 volts dropped across them.

$$I_{R2} = \frac{E_{R2}}{R2} = \frac{27\ V}{18\ k\Omega}$$

$$I_{R2} = 1.5\text{ mA}$$



Since R_2 and R_3 have the same resistance value and the same voltage across them, they have the same current flow through them. You could have found the current through R_2 and R_3 by simply realizing that they must divide the total current of 3 milliamps equally between them.

$$I_{R2} = I_{R3} = \frac{I_T}{2} = \frac{3\ mA}{2}$$

If R_2 and R_3 did not have the same resistance value, you could have found the current through R_3 by subtraction. You know the total current and you know the current through R_2, so the remainder of the current must flow through R_3.

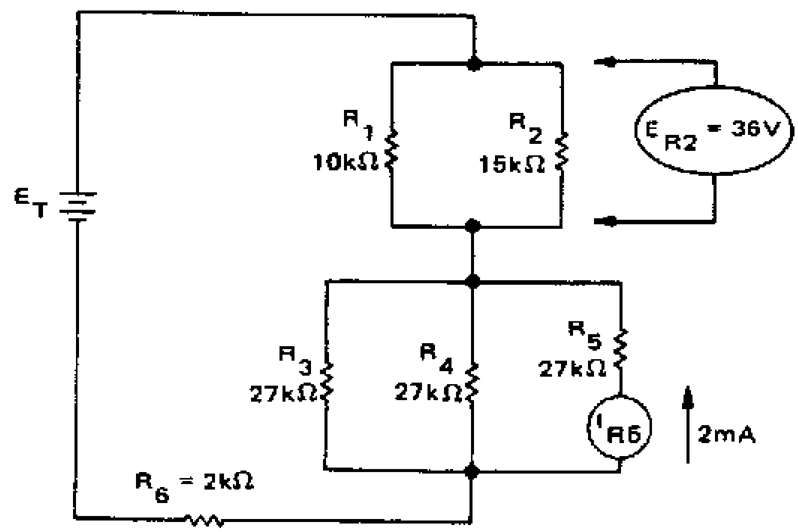
$$I_{R3} = I_T - I_{R2}$$

$$I_{R3} = 3\text{ mA} - 1.5\text{ mA}$$

$$I_{R3} = 1.5\text{ mA}$$

and the circuit is completely solved.

2. In the series-parallel circuit shown, calculate the total equivalent resistance and all unknown voltages and currents using Ohm's law and circuit reduction techniques.



In order to keep track of all the knowns and unknowns, make a chart as shown on the next page and fill in the known values. Then you can fill in the unknown values as you calculate them.

E	I	R
E_{R1} = 36 V	I_{R1} =	R_1 = 10 kΩ
E_{R2} = 36 V	I_{R2} =	R_2 = 15 kΩ
E_{R3} =	I_{R3} =	R_3 = 27 kΩ
E_{R4} =	I_{R4} =	R_4 = 27 kΩ
E_{R5} =	I_{R5} = 2 mA	R_5 = 27 kΩ
E_{R6} =	I_{R6} =	R_6 = 2 kΩ
E_T =	I_T =	R_T =

Notice that since R_1 and R_2 are in parallel, the voltage across them is the same.

You can use Ohm's law in the form I = E/R to calculate I_{R1} and I_{R2}.

$$I_{R1} = \frac{E_{R1}}{R_1} \qquad I_{R2} = \frac{E_{R2}}{R_2}$$

$I_{R1} = \frac{36\ V}{10\ k\Omega}$ $I_{R2} = \frac{E_{R2}}{R_2}$

$I_{R1} = 3.6$ mA $I_{R2} = 2.4$ mA

You know that the total current in a parallel circuit equals the sum of the individual branch currents. In this circuit, the total current flows through the combination of R_1 and R_2; you can add I_{R1} and I_{R2} to get I_T.

$$I_T = I_{R1} + I_{R2}$$

$$I_T = 3.6 \text{ mA} + 2.4 \text{ mA}$$

$$I_T = 6.0 \text{ mA}$$

You can now fill in these calculated values on the chart as shown.

E	I	R
$E_{R1} = 36$ V	$I_{R1} = 3.6$ mA	$R_1 = 10$ kΩ
$E_{R2} = 36$ V	$I_{R2} = 2.4$ mA	$R_2 = 15$ kΩ
E_{R3}	I_{R3}	$R_3 = 27$ kΩ
E_{R4}	I_{R4}	$R_4 = 27$ kΩ
E_{R5}	$I_{R5} = 2$ mA	$R_5 = 27$ kΩ
E_{R6}	I_{R6}	$R_6 = 2$ kΩ
E_T	$I_T = 6.0$ mA	R_T

Looking at the chart or the circuit, you can see that you know two things about R_5, you know its resistance, and you know the current flow through it. You can use Ohm's law in the form E = I X R to find E_{R5}.

$$E_{R5} = I_{R5} \text{ X } R_5$$

$$E_{R5} = 2 \text{ mA X } 27 \text{ k}\Omega$$

$$E_{R5} = 54 \text{ V}$$

Because R_3, R_4, and R_5 are in parallel, they have 54 volts dropped across them. If they all have the same voltage across them and they all have the same resistance value, then the current must be the same through all of them. Since I_{R5} equals 2 milliamps, the I_{R3} and I_{R4} also equal 2 milliamps each.

E	I	R
E_{R1} = 36 V	I_{R1} = 3.6 mA	R_1 = 10 kΩ
E_{R2} = 36 V	I_{R2} = 2.4 mA	R_2 = 15 kΩ
E_{R3} = 54 V	I_{R3} = 2 mA	R_3 = 27 kΩ
E_{R4} = 54 V	I_{R4} = 2 mA	R_4 = 27 kΩ
E_{R5} = 54 V	I_{R5} = 2 mA	R_5 = 27 kΩ
E_{R6}	I_{R6}	R_6 = 2 kΩ
E_T	I_T = 6.0 mA	R_T

You could check your work at this point by adding I_{R3}, I_{R4} and I_{R5} to see that they do add up to the total current of 6 milliamps.

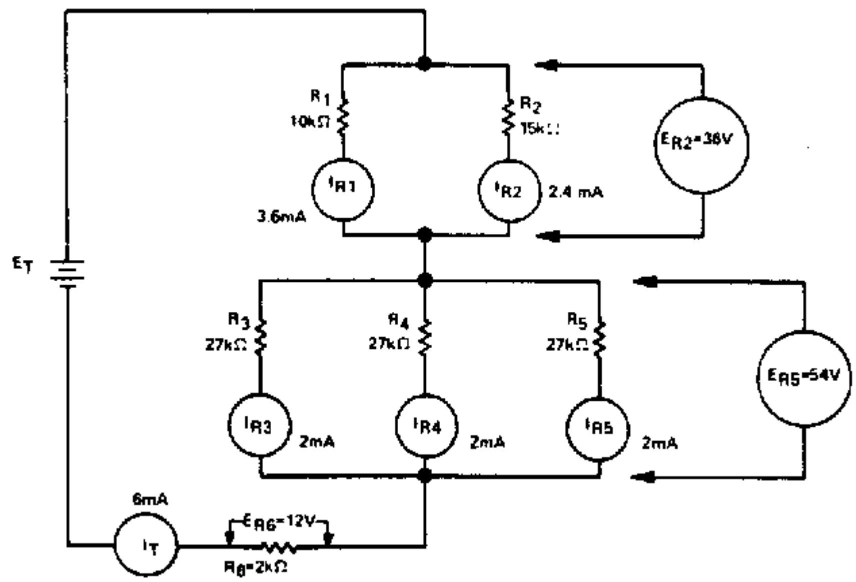
Because R_6 is in series with the rest of the circuit, the total current must flow through it. Thus I_{R6} equals 6 milliamps and you can now use this information to find E_{R6}.

$$E_{R6} = I_{R6} \times R_6$$

$$E_{R6} = 6 \text{ mA} \times 2 \text{ k}\Omega$$

$$E_{R6} = 12 \text{ V}$$

As shown, you know the voltage across and current flow through each portion of the circuit.



The voltage across R_1 and R_2 is the same; $E_{R1,2}$ equals 36 volts. The voltage is also the same across R_3, R_4, and R_5; $E_{R3,4,5}$ equals 54 volts. You also know the voltage across R_6; E_{R6} equals 12 volts. From series circuit laws, these voltages can be added to find the total voltage applied to the circuit.

$$E_T = E_{R1,2} + E_{R3,4,5} + E_{R6}$$

$$E_T = 36 \text{ V} + 54 \text{ V} + 12 \text{ V}$$

$$E_T = 102 \text{ V}$$

The only unknown quantity remaining to be calculated is the total resistance. This can be found in either of two ways. One way is to use Ohm's law in the form:

$$R_T = \frac{E_T}{I_T}$$

When you substitute the appropriate values in the formula, you obtain:

$$R_T = \frac{102\ V}{6\ mA}$$

$$R_T = 17\ k\Omega$$

Circuit reduction techniques can also be used to find R_T. First, consider R_1 in parallel with R_2. Using the product-over-the-sum formula:

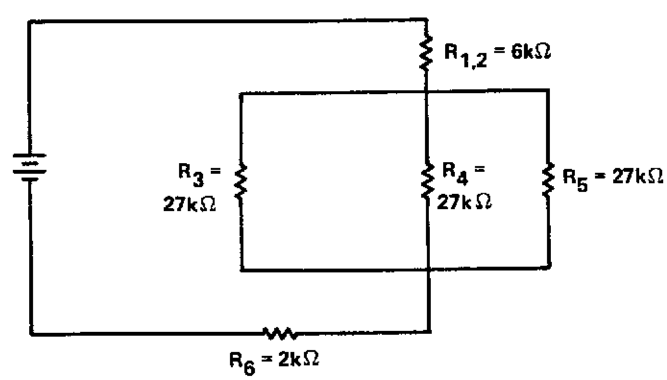
$$R_{1,2} = \frac{R_1\ X\ R_2}{R_1\ +\ R_2}$$

$$R_{1,2} = \frac{10\ k\Omega\ X\ 15\ k\Omega}{10\ k\Omega\ +\ 15\ k\Omega}$$

$$R_{1,2} = \frac{(1\ X\ 10^{+4})\ X\ (1.5\ X\ 10^{+4})}{(1\ X\ 10^{+4})\ +\ (1.5\ X\ 10^{+4})}$$

$$R_{1,2} = \frac{1.5\ X\ 10^{+8}}{2.5\ +\ 10^{+4}}$$

$$R_{1,2} = 0.6\ X\ 10^{+4} = 6\ k\Omega$$

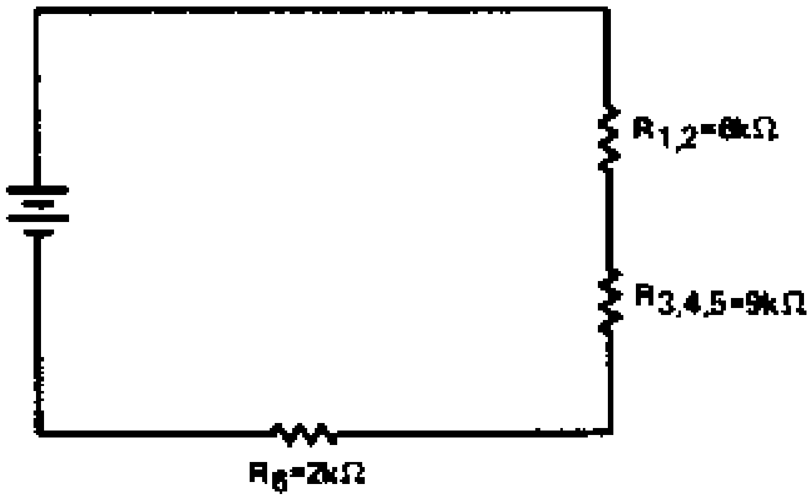


Because R_3, R_4 and R_5 all have the same resistance value, they can be reduced to an equivalent resistance by using the formula:

$$R_{eq} = \frac{R_S}{N}$$

$$R_{eq} = \frac{27\ k\Omega}{3}$$

$$R_{eq} = 9\ k\Omega$$



These three resistance are now in series and can be added to find R_T.

$$R_T = R_{1,2} + R_{3,4,5} + R_6$$

$$R_T = 6\ k\Omega + 9\ k\Omega + 2\ k\Omega$$

$$R_T = 17\ k\Omega$$

and this agrees with the previous calculation.

The chart can be filled in as shown, and the circuit is completely solved.

E	I	R
E_{R1} = 36 V	I_{R1} = 3.6 mA	R_1 = 10 kΩ
E_{R2} = 36 V	I_{R2} = 2.4 mA	R_2 = 15 kΩ
E_{R3} = 54 V	I_{R3} = 2 mA	R_3 = 27 kΩ
E_{R4} = 54 V	I_{R4} = 2 mA	R_4 = 27 kΩ
E_{R5} = 54 V	I_{R5} = 2 mA	R_5 = 27 kΩ
E_{R6} = 12 V	I_{R6} = 6 mA	R_6 = 2 kΩ
E_T = 102 V	I_T = 6 mA	R_T = 17 kΩ

The key objective of this lesson has been achieved if you can analyze any series parallel circuit in a variety of situations such as:

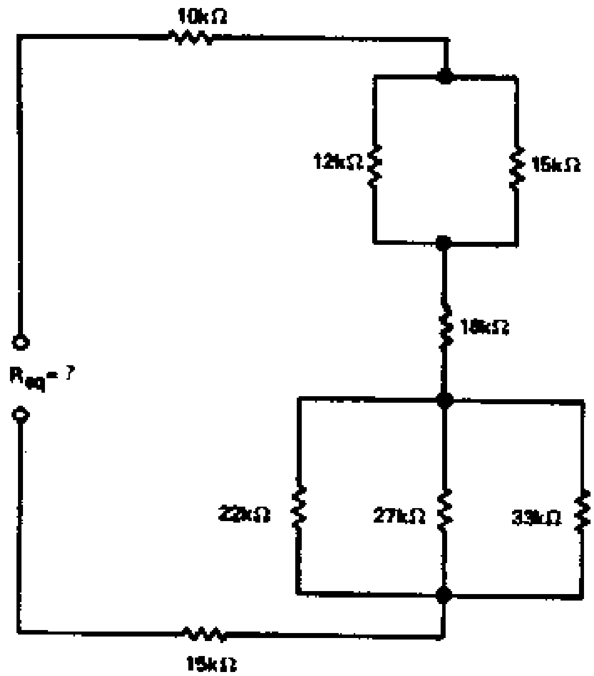
1. Given a series-parallel wired network of resistors, calculate their equivalent resistance, R_{eq}.
2. Given a series-parallel circuit with all of the resistor values and the applied voltage labeled, calculate any or all of the voltages across and currents through each resistor, as well as the total circuit current and equivalent resistance.
3. Given a series-parallel circuit schematic with several known values labeled, calculate any unknown values required.

The practice problems that follow are designed to give you as much practice as you may need in these areas. It is suggested that you work enough of these to enable you to approach and analyze any series-parallel circuit without referring back to the lesson.

Depending upon the approach you use in solving these problems and how you round off intermediate results, your answers may vary slightly from those given here. However, any differences you encounter should only occur in the third significant digit of your answer. If the first two significant digits of your answers do not agree with those given here, recheck your calculations.

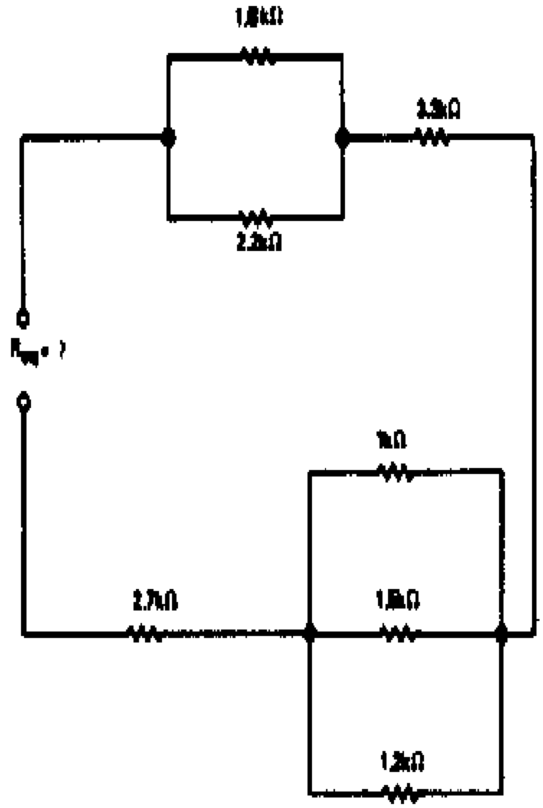
Problems

1. Find Req for the following circuits.



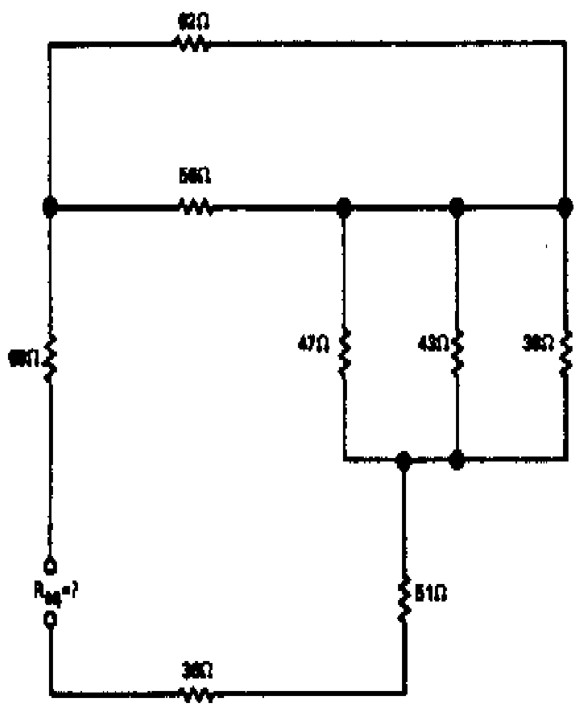
R_{eq} = ____________

2.



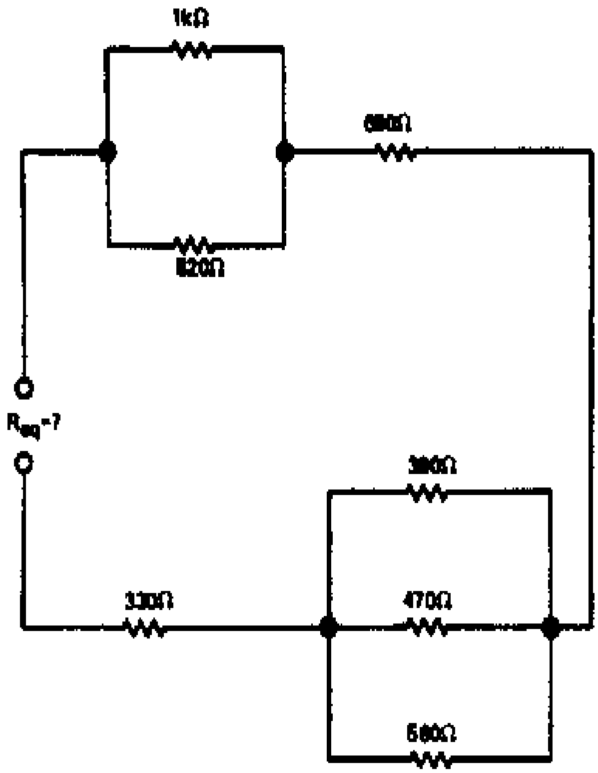
R_{eq} = ________

3.



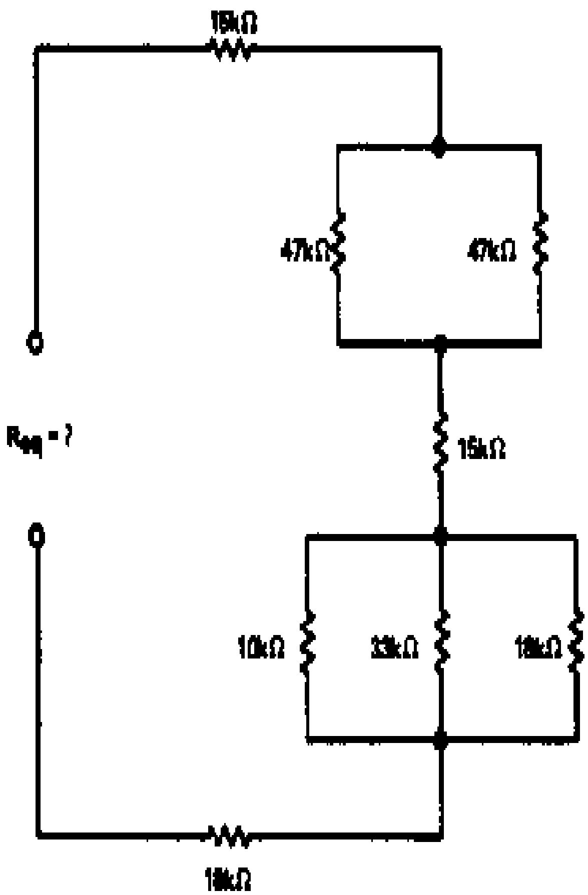
R_{eq} = ________

4.



R_{eq} = __________

5.



R_{eq} = ________

Answers

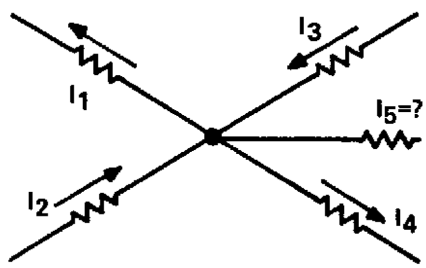
1. $R_{eq} = 58.5\ k\Omega$
2. $R_{eq} = 7.39\ k\Omega$
3. $R_{eq} = 199\ \Omega$
4. $R_{eq} = 1.61\ k\Omega$
5. $R_{eq} = 76.9\ k\Omega$

[redacted]

Introduction to Kirchhoff's Law

Worked Through Examples

1. Write a node equation for the diagram shown below, substitute the appropriate currents and solve the equation for I_5. Also indicate the direction of I_5.



$I_1 = 2$ A
$I_2 = 3.5$ A
$I_3 = 4$ A
$I_4 = 2.5$ A

From Kirchhoff's current law you know that whatever current arrives at a junction must equal the current that leaves the junction. Write down the currents entering the junction on one side of an equals sign, and then write down the currents that leave the junction on the other side of the equals sign.

$$\text{Leaving} = \text{Entering}$$

$$I_1 + I_4 = I_2 + I_3$$

On which side of the equals sign does I_5 belong? If you substitute the values for I_1 through I_4 in the equation, you will see.

$$\text{Leaving} = \text{Entering}$$

$$2\text{ A} + 2.5\text{ A} = 3.5\text{ A} + 4\text{ A}$$

$$4.5\text{ A} = 7.5\text{ A}$$

Obviously, 4.5 amps does not equal 7.5 amps, so I_5 must belong with the 4.5 amp leaving the junction.

$$\text{Leaving} = \text{Entering}$$

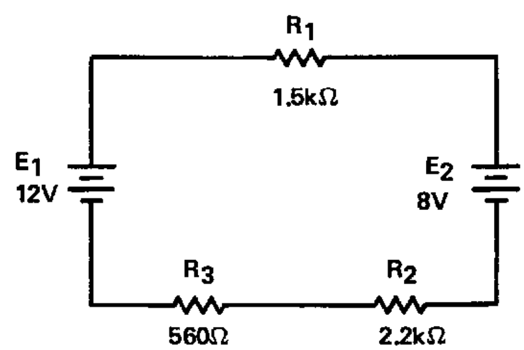
$$4.5\text{ A} + I_5 = 7.5\text{ A}$$

In order for the currents leaving to equal the currents entering, I_5 must be the right value so that there will be 7.5 amps leaving *and* entering the junction. I_5 should be 3 amps *leaving* the junction. You can prove this by subtracting 4.5 amps from each side of the equation.

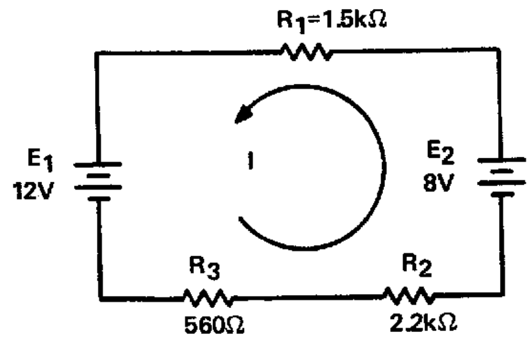
$$\begin{array}{rcr} 4.5\text{ A} + I_5 & = & 7.5\text{ A} \\ \underline{-4.5\text{ A} \qquad} & & \underline{-4.5\text{ A}} \\ I_5 & = & 3\text{A} \end{array}$$

Thus, I_5 does equal 3 amps and it must leave the junction.

2. Write a loop equation for the circuit shown below using electron current, and write another loop equation using conventional current.



Step One: Assign a current direction. Any direction is fine but more than likely the actual direction of electron current is counterclockwise since E_1 is larger than E_2. Assume that the electron current is flowing in the counterclockwise direction and label it accordingly.



Step Two: Traverse the circuit and write down all the source voltages and IR voltages according to the rules presented in this lesson. If you start at the positive terminal of E_1 and move through the circuit counterclockwise, you should get:

$+E_1$ (since you go through E_1 in the *same* direction it pushes electron current)

$-IR_3$ (since you traverse R_3 *in* the direction of electron current)

$-IR_2$ (since you traverse R_2 *in* the direction of electron current)

$-E_2$ (since you go through E_2 *against* the direction it is pushing electron current)

$-IR_1$ (Since you traverse R_1 *in* the direction of electron current).

When you set this equal to zero, the loop equation for this circuit, considering electron current, is:

$$E_1 - IR_3 - IR_2 - E_2 - IR_1 = 0$$

To write a loop equation for conventional current, traverse the loop again and write down the voltages according to your rules. Assume the same direction for current as before. If you start at the same point (the positive terminal of E1) and mover through the circuit counterclockwise, you should get:

$-E_1$ (since you go through E_1 *against* the direction it pushes conventional current)

$-IR_3$ (since you traverse R_3 *in* the assumed direction for conventional current)

$-IR_2$ (since you traverse R_2 *in* the assumed direction for conventional current)

$+E_2$ (since you go through E_2 in the *same* direction it pushes conventional current)

$-IR_1$ (Since you traverse R_1 *in* the assumed direction for conventional current).

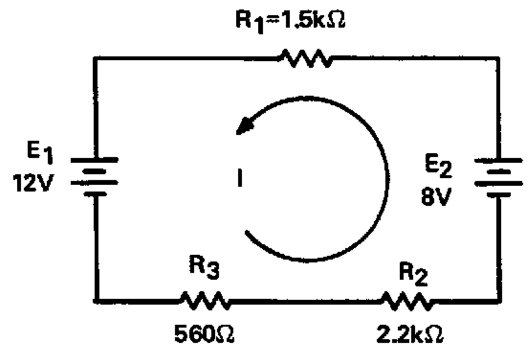
This loop equation for this circuit, considering conventional current, is:

$$-E_1 - IR_3 - IR_2 + E_2 - IR_1 = 0$$

3. Solve each of the equations from the previous example for the current.

   Electron Current Equation

   $E_1 - IR_3 - IR_2 - E_2 - IR_1 = 0$



   First, substitute the appropriate values from the circuit into the equation.

   $$12 - 0.56\ kI - 2.2\ kI - 8 - 1.5\ kI = 0$$

   When the two source voltages are added algebraically, they yield 4.

   $$4 - 0.56\ kI - 2.2\ kI - 1.5\ kI = 0$$

   You can combine the I terms to get:

   $$4 - 4.26\ kI = 0$$

   Transpose the 4, remembering to change its sign.

   $$-4.26\ kI = -4$$

   Divide both sides of the equation by -4.26 k.

   $$\frac{-4.26\ kI}{-4.26\ k} = \frac{-4}{-4.26\ k}$$

   $$I = 0.939\ mA \text{ or } 939\ \mu A$$

   Since this answer is positive, the assumed direction for the electron current (counterclockwise) is correct.

   Conventional Current Equation

   $$-E_1 - IR_3 - IR_2 + E_2 - IR_1 = 0$$

   First, substitute the appropriate values from the circuit into the equation.

$$- 12 - 0.56\ kI - 2.2\ kI + 8 - 1.5\ kI = 0$$

When the two source voltages are added algebraically, they yield -4. This, as you will see, will make a difference in your answer.

$$- 4 - 0.56\ kI - 2.2\ kI - 1.5\ kI = 0$$

Combine the I terms to get:

$$- 4 - 4.26\ kI = 0$$

Transpose the 4, remembering to change its sign.

$$- 4.26\ kI = 4$$

Divide both sides of the equation by -4.26 k.

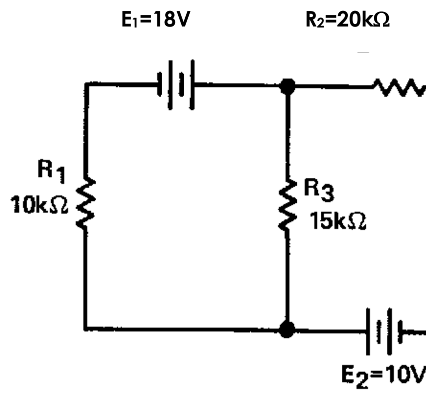
$$\frac{-4.26\ kI}{-4.26\ k} = \frac{4}{-4.26\ k}$$

$$I = -0.939\ mA \text{ or } -939\ \mu A$$

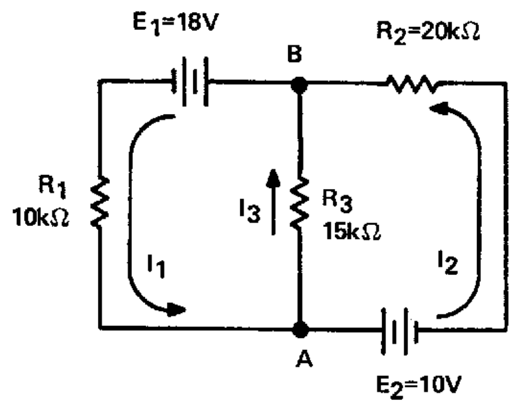
Since this answer is *negative* the assumed direction for the conventional current was wrong, and so you know that the conventional current is actually flowing *clockwise.*

You know, if you thought about this answer for a minute, it makes a great deal of sense. The solution to the electron current equation told you that the electron current was flowing *counterclockwise.* Recall that electron and conventional current have the *same* effect in a circuit; they just flow in *opposite* directions. Thus, you know that conventional current for this circuit must flow in the *clockwise* direction.

4. Write the loop and node equations for the following circuit using electron current. Then solve the equations for the branch currents, including their directions, and use these currents to find the voltage drop across each resistor. Also indicate the polarity of each voltage drop.



First Step: Assign a direction for each current and label it accordingly.



Immediately, you can see from Kirchhoff's current law that at junction point A:

$$I_1 = I_3 + I_2$$

Second Step: Traverse each loop and write down all the voltages you encounter with their correct signs.

Loop 1	$18 - 10\ kI_1 - 15\ kI_3 = 0$ (counterclockwise from point B)
Loop 2	$10 - 20kI_2 + 15\ kI_3 = 0$ (counterclockwise from point A)

Third Step: Simplify the equations. If you substitute $I_3 + I_2$ for I_1 in the first equation, you will then have only two unknowns, and you will have two equations with which to find the two unknowns.

$$18 - 10\ k\ (I_3 + I_2) - 15\ kI_3 = 0$$

$$10 - 20\ kI_2 + 15\ kI_3 = 0$$

In the first equation, multiply I_3 and I_2 by - 10k.

$$18 - 10\ kI_3 - 10\ kI_2 -15\ kI_3 = 0$$

You can now combine the I_3 terms.

$$18 - 10\ kI_2 - 25\ kI_3 = 0$$

If you multiply both sides of this equation by -2, you can then add it to your equation for loop 2.

$$(-2)\ (18 - 10\ kI_2 - 25\ kI_3) = (0)\ (-2)$$

$$-36 + 20\ kI_2 + 50\ kI_3 = 0$$

Fourth Step: Add the equations to eliminate one of the unknown currents, thus enabling you to calculate the other current.

$$-36 + 20\ kI_2 + 50\ kI_3 = 0$$

$$\underline{10 - 20\ kI_2 + 15\ kI_3 = 0}$$

$$-26 \qquad\qquad + 65\ kI_3 = 0$$

$$65\ kI_3 = 26$$

$$I_3 = \frac{26}{65\ k} = 0.4\text{ mA} = 400\ \mu\text{A}$$

Since this answer is positive, you know that the assumed direction for I_3 is correct.

Fifth Step: Substitute the value of I_3 in one of the previous loop equations to find I_1 or I_2.

$$\text{Loop 2} \qquad 10 - 20\ kI_2 + 15\ kI_3 = 0$$

$$10 - 20\ kI_2 + 15\ k\ (0.4\text{ mA}) = 0$$

When 15 k is multiplied by 4 mA, the result is 6, which can then be added to the 10.

$$10 - 20\ kI_2 + 6 = 0$$

$$16 - 20\ kI_2 = 0$$

Transpose and divide.

$$-20\ kI_2 = -16$$

$$\frac{-20\ kI_2}{-20\ k} = \frac{-16}{-20\ k}$$

$$I_2 = 0.8\ mA = 800\ \mu A$$

This answer is also positive, so the assumed direction for I_2 is correct.

Sixth Step: Substitute I_2 and I_3 in the node current equation to find I_1.

$$I_1 = I_3 + I_2$$

$$I_1 = 0.4\ mA + 0.8\ mA$$

$$I_1 = 1.2\ mA$$

Seventh Step: Use Ohm's law to calculate the voltage drops across the resistors.

$$E_{R1} = I_1 \times R_1$$

$$E_{R1} = 1.2\ mA \times 10\ k\Omega$$

$$E_{R1} = 12\ V$$

$$E_{R2} = I_2 \times R_2$$

$$E_{R2} = 0.8\ mA \times 20\ k\Omega$$

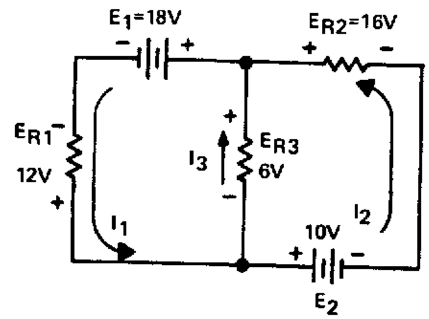
$$E_{R2} = 16\ V$$

$$E_{R3} = I_3 \times R_3$$

$$E_{R3} = 0.4\ mA \times 15\ k\Omega$$

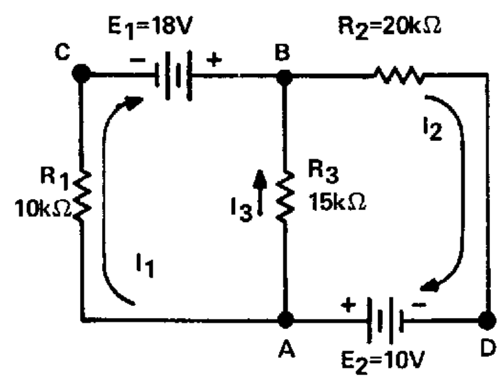
$$E_{R3} = 6\ V$$

Recall the rule for determining the polarity of the voltage across a resistor, which states that electron current flows through a resistor from minus to plus or from the negative side to the positive side. Thus, the voltage drops and their polarities are as shown on the next page.



5. Write the loop and node equations for the circuit shown in example 4 using *conventional* current. Then solve the equations for the branch currents, including their directions. Also indicate the polarities of the voltage drops produced by these conventional currents.

   First Step: Assign a direction for each current and label it accordingly.



Then, from Kirchhoff's current law, the node current equation for node A is:

$$I_2 = I_3 + I_1$$

Second Step: Traverse each loop and write down all the voltages you encounter with their correct signs.

Loop 1	$18 + 15\ kI_3 - 10\ kI_1 = 0$ (clockwise from point C)
Loop 2	$10 - 15\ kI_3 - 20\ kI_2 = 0$ (clockwise from point D)

Third Step: Simplify the equations. If you substitute $I_3 + I_1$ for I_2 in the second equation, you will have two equations with two unknowns. You can than easily solve the equations for the unknown currents.

$$10 - 15\ kI_3 - 20\ kI_2 = 0$$

$$10 - 15\ kI_3 - 20\ k\ (I_3 + I_1) = 0$$

Multiply I_3 and I_1 by - 20 k.

$$10 - 15\ kI_3 - 20\ kI_3 - 20\ kI_1 = 0$$

You can combine the I_3 terms.

$$10 - 35\ kI_3 - 20\ kI_1 = 0$$

If you divide both sides of this equation by -2, you can add it to the equation for loop 1.

$$(10 - 35\ kI_3 - 20\ kI_1) \div (-2) = (0) \div (-2)$$

$$-5 + 17.5\ kI_3 + 10\ kI_1 = 0$$

Fourth Step: Add the equations to eliminate one of the unknown currents, thus enabling you to find the other current.

Loop 1	$18 + 15\ kI_3 - 10\ kI_1 = 0$
Loop 2	$\underline{-5 + 17.5\ kI_3 + 10\ kI_1 = 0}$
	$13 + 32.5\ kI_3 \qquad = 0$
	$32.5\ kI_3 = -13$
	$I_3 = \dfrac{-13}{32.5\ k}$
	$I_3 = -0.4\ mA = -\ 400\ \mu A$

Since this answer is negative, you know that the assumed direction for I_3 is wrong and that the conventional current I_3 actually flows down through R_3.

Fifth Step: Substitute the value of I_3 in one of the previous loop equations to find I_1 or I_2.

Loop 1	$18 + 15\ kI_3 - 10\ kI_1 = 0$
	$18 + 15\ kI_3\ (-0.4\ mA) - 10\ kI_1 = 0$

When 15 k is multiplied by -0.4 mA, the result is -6, which can then be added algebraically to the 18.

$$18 - 6 - 10\ kI_1 = 0$$

$$12 - 10\ kI_1 = 0$$

Transpose and divide.

$$-10\,kI_1 = -12$$

$$\frac{10kI}{-10k} = \frac{-12}{-10k}$$

$$I_1 = 1.2\text{ mA}$$

This answer is positive, so you know that the assumed direction for I_1 is correct.

Sixth Step: Substitute I_1 and I_3 in the node current equation to find I_2:

$$I_2 = I_3 + I_1$$

$$I_2 = -0.4\text{ mA} + 1.2\text{ mA}$$

$$I_2 = 0.8\text{ mA or } 800\ \mu\text{A}$$

The answer is positive so the assumed direction for I_2 is correct.

Seventh Step: Use Ohm's law to find the voltage drops across the resistors. Since the answers for the currents have the same numerical value as in the previous example, the voltage drops will be the same as they were before, or:

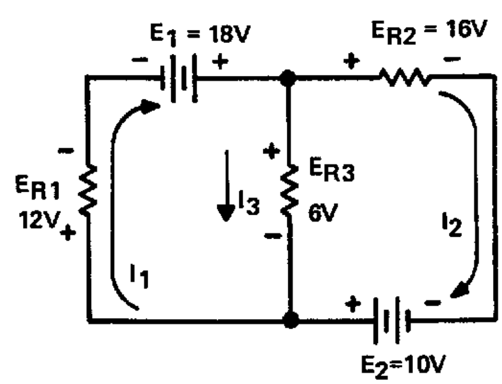
$$E_{R1} = 12\text{ V}$$

$$E_{R2} = 16\text{ V}$$

$$E_{R3} = 6\text{ V}$$

In determining the correct polarities of these voltage drops, remember two things:

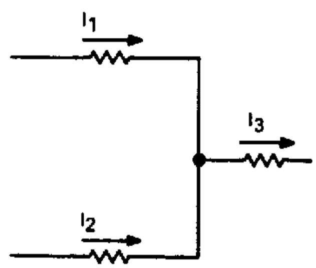
1. Conventional current flows through resistors from plus to minus.
2. I_3 is actually flowing down through R_3.



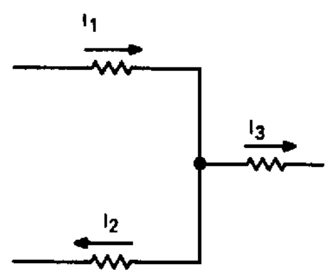
Depending upon the approach you use in solving these problems and how you round off intermediate results, your answers may vary slightly from those given here. However, any differences you encounter should only occur in the third significant digit of your answer. If the first two significant digits of your answers do not agree with those given here, recheck your calculations.

1. Write the node equations for the following diagrams.

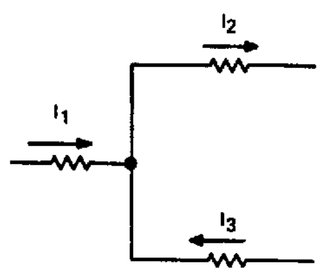
a.



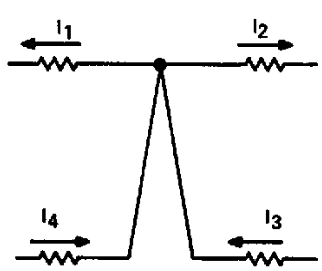
b.



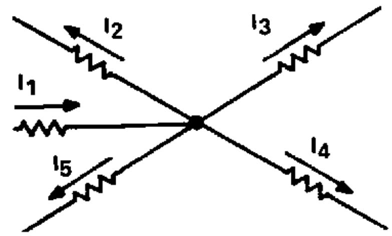
c.



d.

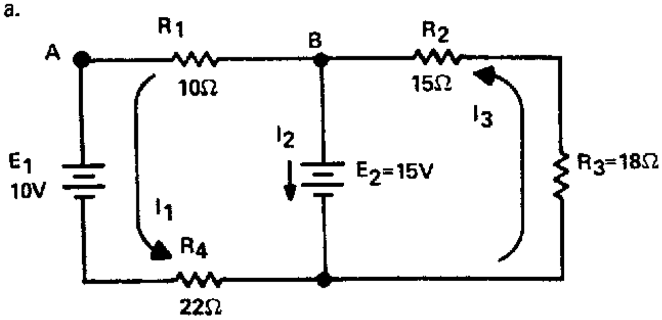


e.

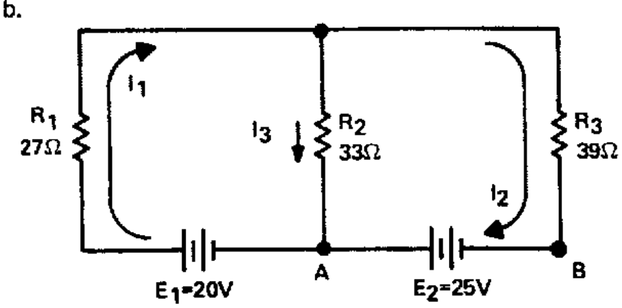


2. Write the loop equations for the following diagrams.

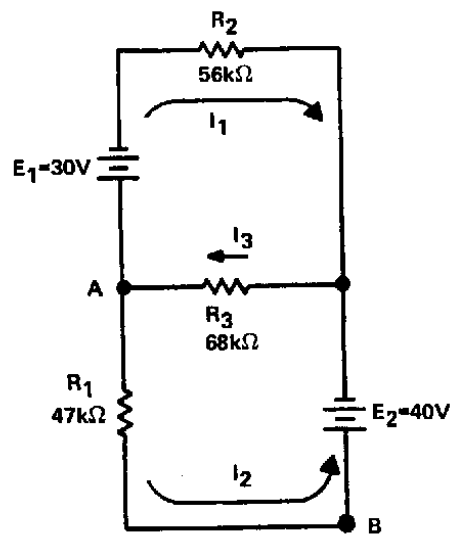
a.



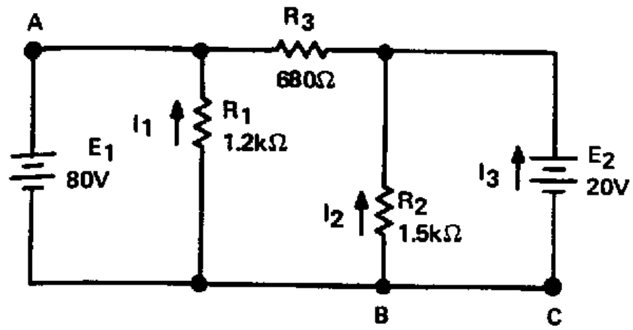
b.



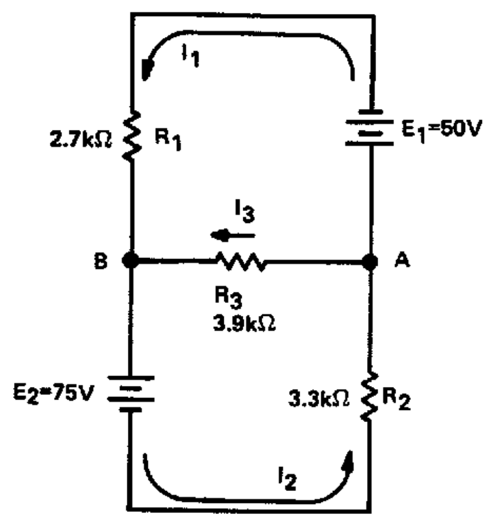
c.



d.

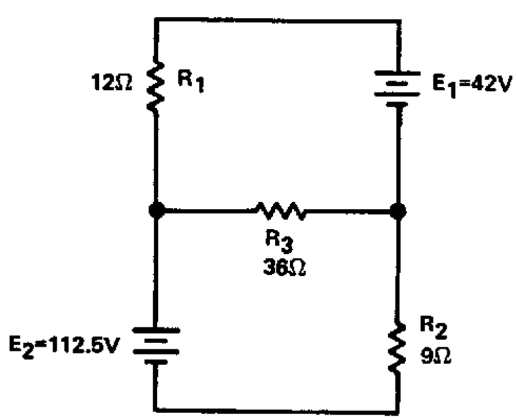


e.

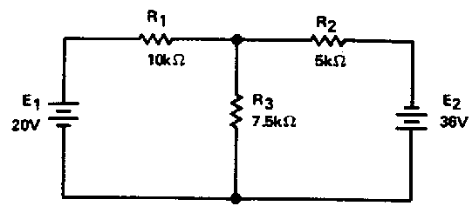


3. Solve the following circuits for all currents and voltage drops. Indicate the polarity of the voltage drops and the direction of the currents.

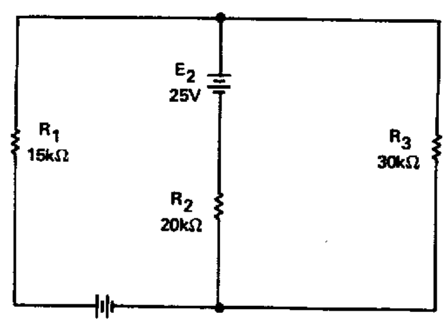
a.



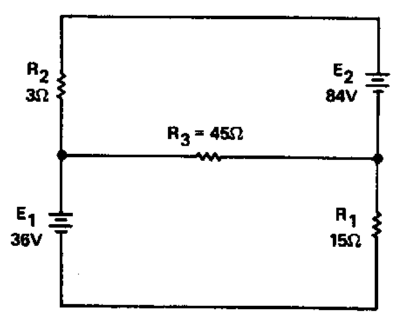
b.


R1
10kΩ
R2
5kΩ
E1
20V
R3
7.5kΩ
E2
36V

c.


E2
25V
R1
15kΩ
R3
30kΩ
R2
20kΩ
E1 = 82.5V

d.


R2
3Ω
E2
84V
R3 = 45Ω
E1
36V
R1
15Ω

Answers

1.a. $I_1 + I_2 = I_3$

1.b. $I_1 = I_2 + I_3$

1.c. $I_1 + I_3 = I_2$

1.d. $I_4 + I_3 = I_1 + I_2$

1.e. $I_1 = I_2 + I_3 + I_4 + I_5$

2.a. Loop 1 — Start at point A and trace the loop ccw.

$10 - 22I_1 - 15 - 10I_1 = 0$

Loop 2 — Start at point B and trace the loop ccw.

$15 - 18I_3 - 15I_3 = 0$

2.b. Loop 1 — Start at point A and trace the loop ccw.

$20 - 27I_1 - 33I_3 = 0$

Loop 2 — Start at point B and trace the loop ccw.

$25 + 33I_3 - 39I_2 = 0$

2.c. Loop 1 — Start at point A and trace the loop ccw.

$30 - 56\ kI_1 - 68\ k_{I3}$

Loop 2 — Start at point B and trace the loop ccw.

$40 - 68\ kI_3 - 47I_2$

2.d. Loop 1 — Start at point A and trace the loop ccw.

$80 - 1.2\ kI_1 = 0$

Loop 2 — Start at point B and trace the loop ccw.

$-1.5\ kI_2 - .68\ k\ (I_2 + I_3) + 1.2\ kI_1 = 0$

Loop 3 — Start at point C and trace the loop ccw.

$-20 + 1.5\ kI_2 = 0$

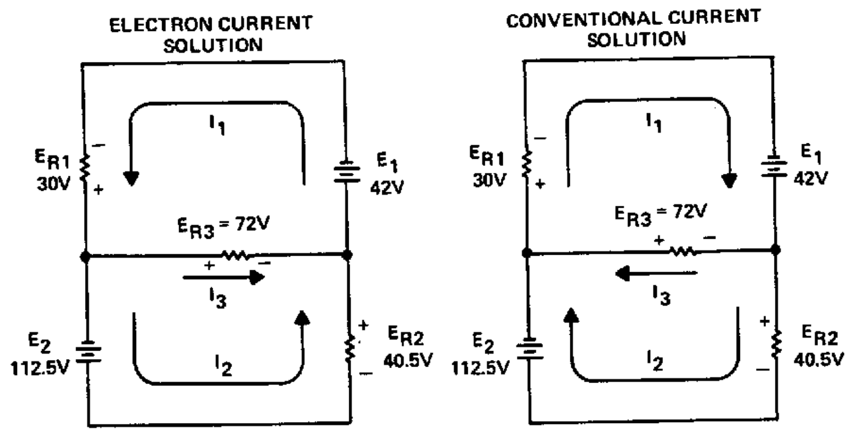
2.e. Loop 1 — Start at point A and trace the loop ccw.

$50 - 2.7\ kI_1 + 3.9\ kI_3 = 0$

Loop 2 — Start at point B and trace the loop ccw.

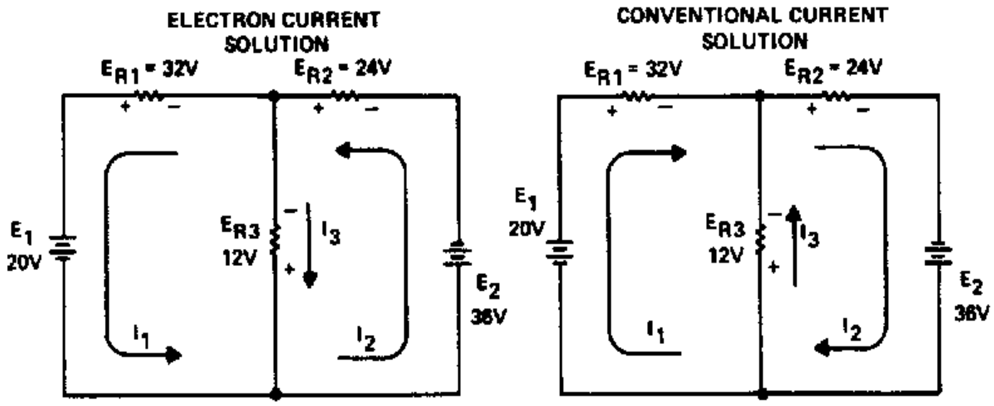
$75 - 3.3\ kI_2 - 3.9\ kI_3 = 0$

3.a.



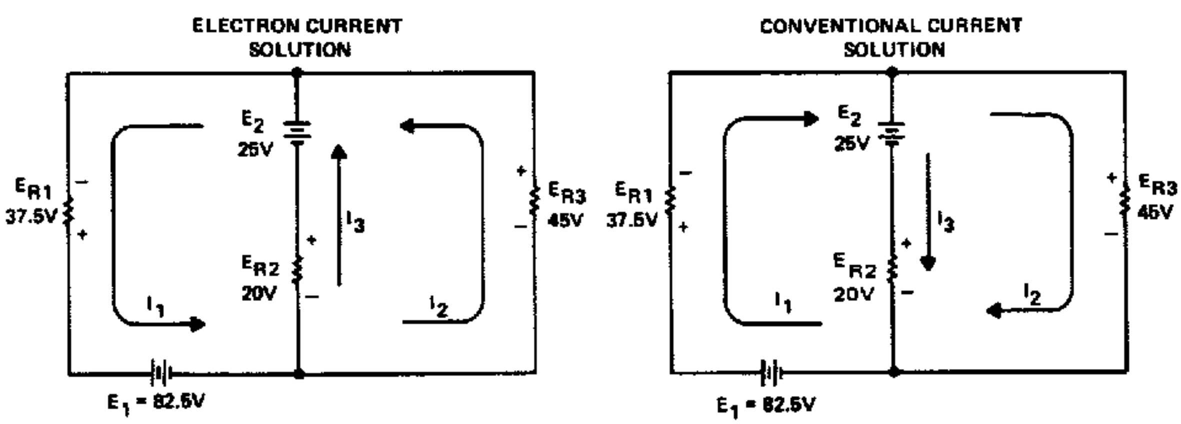
I_1 = 2.5A I_2 = 4.5A I_3 = 2A

3.b.



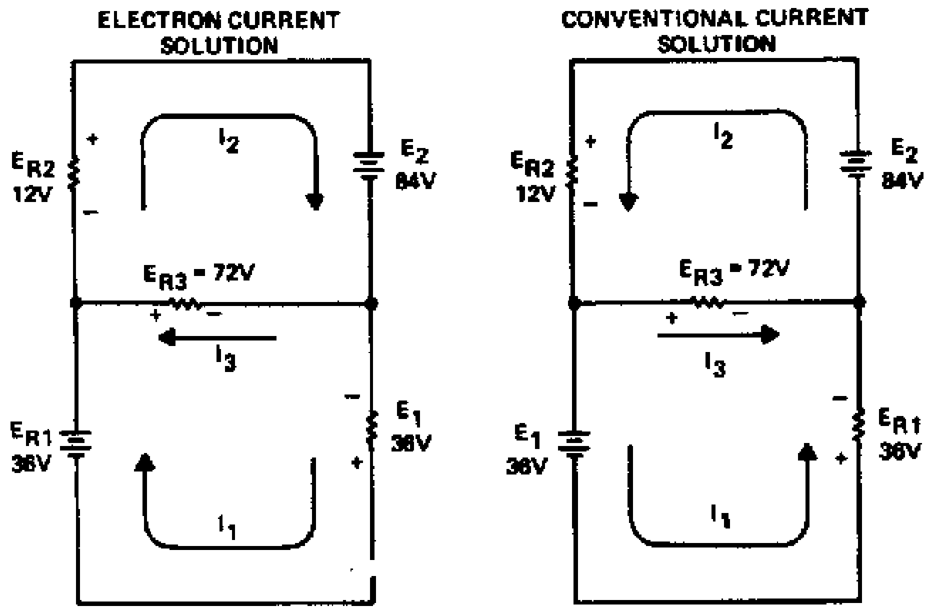
I_1 = 3.2mA I_2 = 4.8mA I_3 = 1.6mA

3.c.



I_1 = 2.5mA I_2 = 1.5mA I_3 = 1.0mA

3.d.



$I_1 = 2.4A \quad I_2 = 4A \quad I_3 = 1.6A$

[redacted]

Capacitors and the RC Time Constant

Worked Through Examples

1. Find the time constant of a circuit containing a 10-kilohm resistor in series with a 0.82-microfarad capacitor.

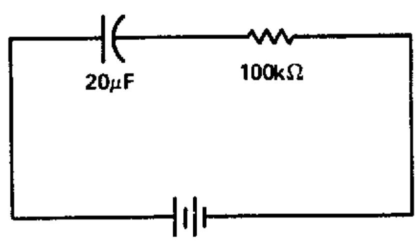
   To solve this problem, you must use the time constant formula T = RC. Substituting in the circuit values, the formula reads T = 10 kΩ X 0.82 μF. In scientific notation the values are: T = 1.0 X 10^4 X 8.2 X $10^{-7.}$

   1.0×10^4

   $\underline{\times\ 8.2 \times 10^{-7}}$

   T = 8.2 X 10^{-3} seconds (s) or 8.2 milliseconds (ms)

2. Find the time constant of this circuit:



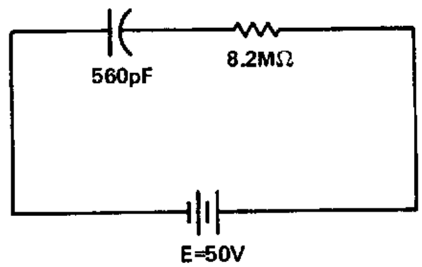
   Use the formula: T = RC. First substitute in the circuit values: R = 100 kΩ, C = 20 μF.

   T = 100 kΩ, C = 20 μF

   T = 1.0 X 10^5 X 2.0 X 10^{-5}

   T = 2.0 seconds

3. How long will it take the capacitor in the following circuit to reach full charge?



First, use the time constant formula T = RC

$$T = RC$$

$$T = 8.2\ M\Omega \ X\ 560\ \rho F$$

$$T = 8.2\ X\ 10^{6}\ X\ 5.6\ X\ 10^{10}$$

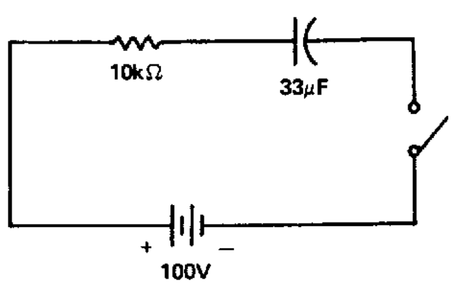
$$T = 4.59\ X\ 10^{-3}\ s \text{ or } 4.59\ ms$$

You must remember that the RC time constant formula you just worked gives you *one* time constant (in seconds). *Five* time constraints are required for full charge. So, multiply the time constant by 5 to arrive at the correct answer.

$$\begin{array}{l} 4.59\ X\ 10^{-3} \\ \underline{X\ \ 5\qquad\qquad} \\ 22.95\ X\ 10^{-3} \text{ or } 2.3\ X\ 10^{-2} \text{ seconds} \end{array}$$

The capacitor will be fully charged after 2.3 X 10^{-2} seconds or 23 milliseconds.

4. Find the voltage across the capacitor in the circuit shown below 500 milliseconds after the switch is closed. (Use the universal time constant graph.)



First, you should calculate the time constant of the circuit.
T = RC

$$T = RC$$

$$T = 10\ \text{k}\Omega,\ X\ 33\ \mu F$$

$$T = 1.0\ X\ 10^{4}\ X\ 3.3\ X\ 10^{-5}$$

$$T = 3.3\ X\ 10^{-1} \text{ or } 330 \text{ ms}$$

Now look at the universal time constant graph. Time (horizontal axis) is measured in time constants. To convert this chart to seconds, multiply 330 milliseconds by each of the time divisions. For example:

1 X 330 ms = 330 ms

1.5 X 330 ms = 495 ms

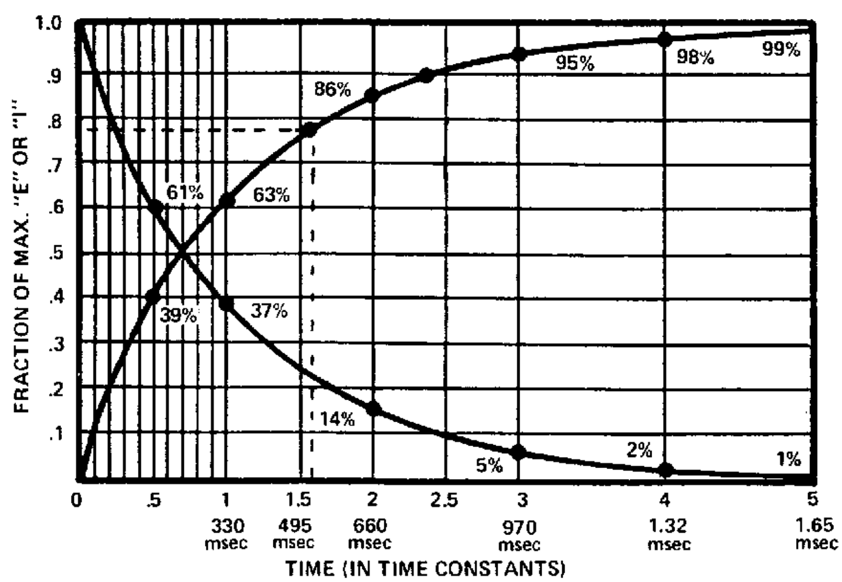
2 X 330 ms = 660 ms

3 X 330 ms = 990 ms

4 X 330 ms = 1.32 s

5 X 330 ms = 1.65 s

Now these values are applied to the universal time constant graph.



Look at the chart and locate the 500 millisecond position on the horizontal axis. Now trace directly upward (following the dotted line) and note the point on the charging curve that is reached at 500 ms.

Tracing to the left from that point, across the graph, you can see that the amplitude at the intersection point is about 0.78 or 78% of the full charge voltage; 0.78 X 100 V. So after 500 ms. the capacitor is charged to 78 volts.

5. Find the charge in coulombs of the capacitor in problem 4, at the end of 500 milliseconds.

   The formula for calculating the charge stored in a capacitor is

$$Q = CE$$

   where

   Q = the stored charge in coulombs

   C = the capacitance in farads

   E = the voltage between the capacitor plates

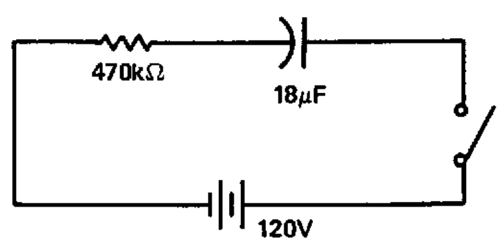
   Substituting the values of capacitance and voltage:

$$Q = 33\ \mu F \times 78\ V$$

$$Q = 3.3 \times 10^{-5} \times 7.8 \times 10^{1}$$

$$Q = 2.57 \times 10^{-3} \text{ coulombs (or 2.57 millicoulombs)}$$

6. Using the universal time constant graph, calculate the time required for the capacitor shown below to charge to 55 volts.



   First, calculate the circuit's time constant using the formula: T = RC

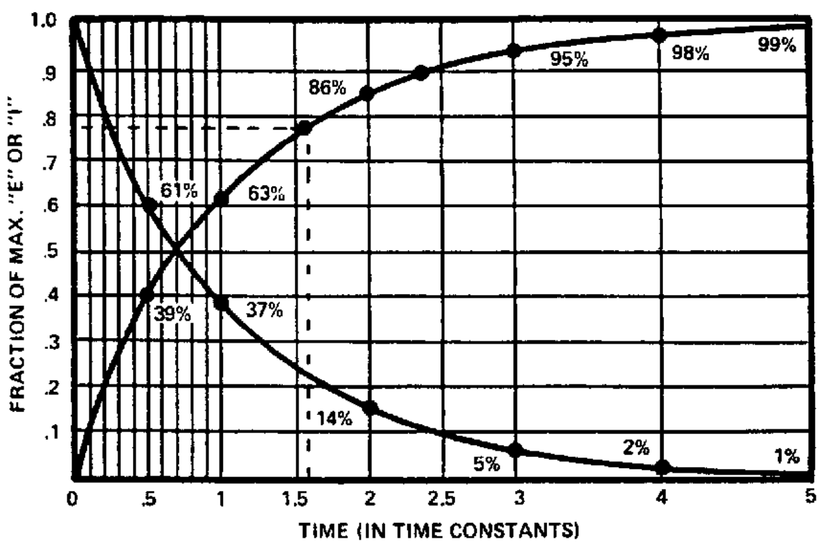
$$T = RC$$

$$T = 470\ k\Omega, \times 18\ \mu F$$

$$T = 4.7 \times 10^{5} \times 1.8 \times 10^{-5}$$

$$T = 8.46\ s$$

Now, the universal time constant curve may be used as follows in solving this problem. First, examine the vertical axis. On this axis the *fraction* of the maximum voltage is located. The *maximum* voltage here is 120 volts: the total applied voltage. What *fraction* of 120 volts is 55 volts? Thus, 55/120 equals 0.458. This is the *fraction* of the applied voltage 55 volts represents. Now, locate 0.458 on the vertical axis of the universal time constant graph. Trace to the right horizontally (a dotted line is drawn in for you to follow) until you intersect the charging curve.



Locate that point on the curve, and then trace directly *down* to the horizontal axis. At this point you read the time elapsed: 0.6 *time constants.* You know that 1 time constant is 8.46 seconds, so the total elapsed time is 0.6 X 8.46 or 5.08 seconds.

7. A "strobe" flash attachment for a camera has a bulb that requires 0.02 coulomb of charge at 450 volts in order to flash properly. What is the minimum size capacitor that could be satisfactorily used?

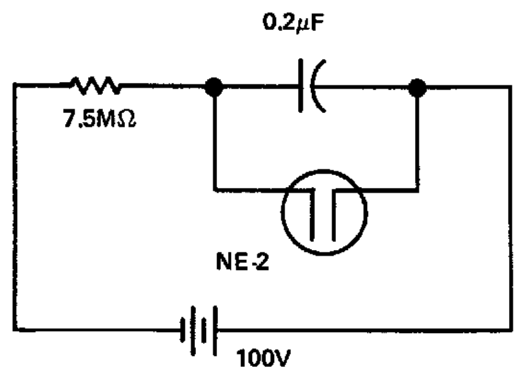
Since both the quantity of charge (Q) and voltage (E) are known, the equation C = Q/E can be used to solve this problem. Simply substitute in the capacitor values and solve for C.

$$C = Q/E$$

$$C = \frac{0.02\ C}{450\ V}\ \underline{(coulomb)}$$

$$C = 0.0000444\ F \text{ or } 44.4\ \mu F$$

8. Find the approximate frequency of oscillation in the circuit shown here.



The circuit shown above is a "relaxation oscillator." It operates on the basis of its RC time constant. The bulb shown connected across the capacitor is an NE-2 neon glow lamp. These lamps require a certain voltage (called the "firing voltage") in order to light. Once lit, the voltage across the lamp must fall significantly below the firing voltage before it will turn "off." Typical "on" and "off" voltages for neon glow lamps are: 75 volts "on" and 50 volts "off." This means that the typical NE-2 will not "light" until the voltage across it reaches 75 volts, but once lit, will continue to glow until the voltage drops below 50 volts. Before the lamp lights, it has a very high resistance (essentially an open circuit). Once the lamp is on, its resistance drops to a low value.

Consider what will happen when one of these lamps is connected across a capacitor as shown in the circuit above. When power is applied to the circuit, the capacitor will begin to charge up to the source voltage. The rate of charging will be controlled by the RC time constant. When the capacitor reaches 75 volts, the neon bulb (which is connected in parallel with the capacitor) will also have 75 volts applied across it. At this instant, the bulb will light, allowing heavy current flow, and thus discharging the capacitor very quickly. As the capacitor discharges, its voltage will drop down below the 50 volts required to keep the neon bulb lit. The bulb goes out and the capacitor again charges up to the 75 volts required to fire the bulb, and the cycle is repeated again and again. As you can see, there are several factors that affect the rate of blinking (or oscillation) of the bulb: the resistor size, the size of the capacitor, the supply voltage, and the characteristics of the individual neon bulb.

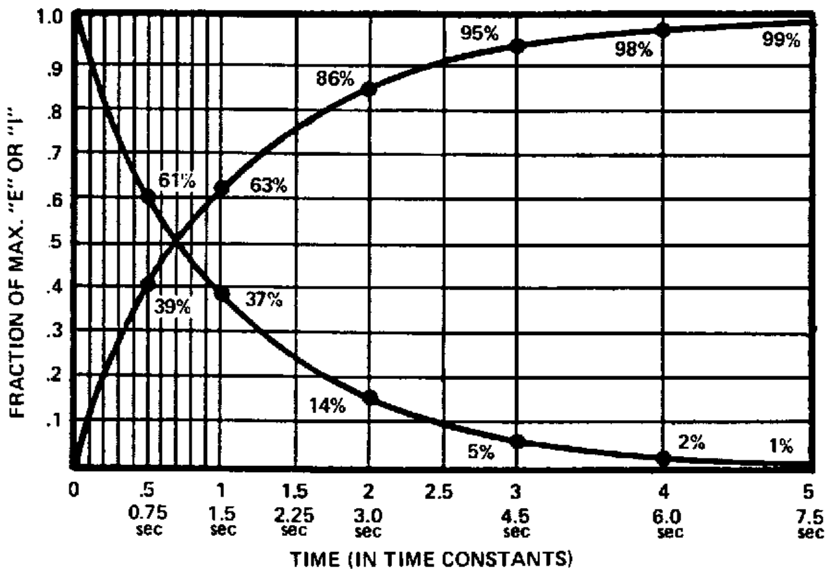
To analyze this problem, first calculate the RC time constant of the circuit and plot it on a universal time constant graph.

$$T = RC$$

$$T = 7.5\ M\Omega,\ X\ 10.2\ \mu F$$

$$T = 7.5\ X\ 10^{6}\ X\ 2.0\ X\ 10^{-7}$$

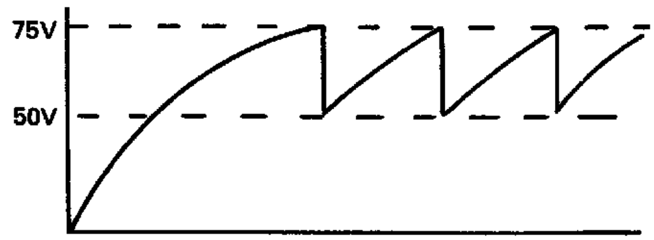
$$T = 1.5\ s$$



1 X 1.5 s = 1.5 s
1.5 X 1.5 s = 2.25 s
2 X 1.5 s = 3.0 s
3 X 1.5 s = 4.5 s
4 X 1.5 s = 6.0 s
5 X 1.5 s = 7.5 s

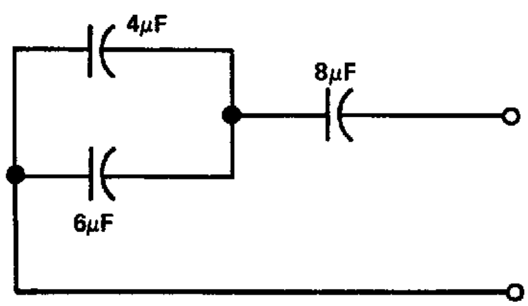
To give a clearer picture of the operation of this circuit, these values are plotted on the horizontal axis of the universal time constant graph above.

The lamp fires at 75 volts, and causes the voltage across the capacitor to rapidly drop to 50 volts so that the lamp then goes out. Voltage across the capacitor, plotted as time goes on, will appear as shown on the next page.



In order to find the time duration between flashes, simply look back at the Universal Time Constant graph you just filled in. Locate 75 volts and 50 volts, and measure the time elapsed between these two points. Seventy-five volts occurs at approximately 1.4 time constants or 2.1 seconds. Fifty volts occurs at 0.7 time constants or 1.055 seconds. The time elapsed is the *difference* between the two times. Subtract and you get 2.1 s - 1.05 s = 1.05 s. So the lamp will blink once every 1.05 seconds. Dividing 60 by 1.05 yields a frequency of 57 flashes per minute.

9. Calculate the total capacitance of this circuit.



Problems of the type shown above give many students headaches because capacitors "add" just the opposite of the way resistors do. Parallel capacitors are added by using a formula similar to the series resistance formula: $C_T = C_1 + C_2 + C_3$ ... Series capacitors must be added by using a formula similar to the parallel resistance formula:

$$C_T = \frac{1}{1/C_1 + 1/C_2 + 1C_{3\ldots}}$$

To solve this problem, the 4-microfarad and the 6-microfarad capacitors should be combined by using the parallel capacitance formula: $C_T = C_1 + C_2 + C_3$ ...

$$C_T = 4\ \mu F + 6\ \mu F$$

$$C_T = 10\ \mu F$$

The 10 microfarads of capacitance must be combined with the 8 microfarads of capacitance by using the series capacitance formula.

$$C_T = \frac{1}{1/C_1 + 1/C_2 + 1C_{3...}}$$

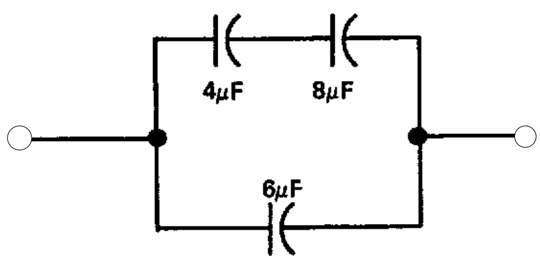
$$C_T = \frac{1}{1/10 + 1/8}$$

$$C_T = \frac{1}{0.1 + 0.125}$$

$$C_T = \frac{1}{0.225}$$

$$C_T = 4.44\ \mu F$$

10. Calculate the total capacitance of the following circuit.



First, find the total capacitance of the upper circuit branch using the series capacitance formula:

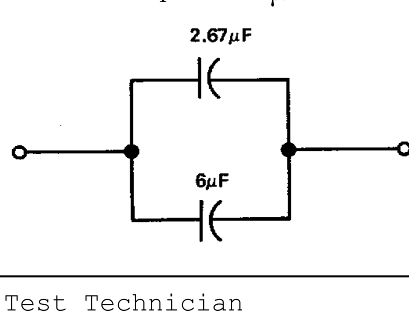
$$C_T = \frac{1}{1/C_1 + 1/C_2 + 1C_{3...}}$$

$$C_T = \frac{1}{1/4 + 1/8}$$

$$C_T = \frac{1}{0.25 + 0.125}$$

$$C_T = \frac{1}{0.375}$$

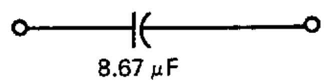
$$C_T = 2.67\ \mu F$$



Now the total capacitance may be found by combining the two parallel capacitances using the parallel capacitance formula $C_T = C_1 + C_2 + C_3 \ldots$

$$C_T = 2.67\ \mu F + 6\ \mu F$$

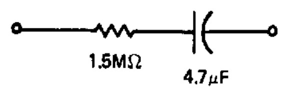
$$C_T = 8.67\ \mu F$$



Depending upon the approach you use in solving these problems and how you round off intermediate results, your answers may vary slightly from those given here. However, any differences you may encounter should only occur in the third significant digit of your answer. If the first two significant digits of your answers do not agree with those given here, recheck your calculations. Answers are on page 336.

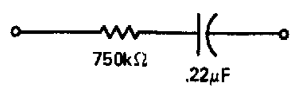
1. Calculate the RC time constant for the following circuits. Fo

a.



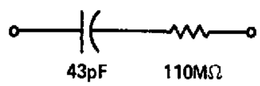
T = ________

b.



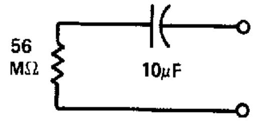
T = ________

c.



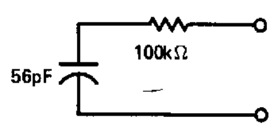
T = ________

d.



T = ________

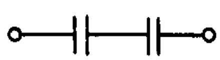
e.



T = ________

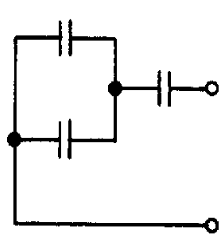
2. Calculate the total capacitance in the following circuits. (All capacitors are 2 μF).

a.



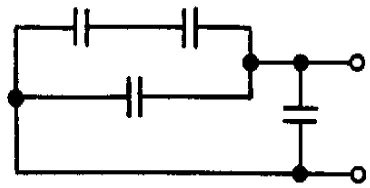
C_T = ________

b.



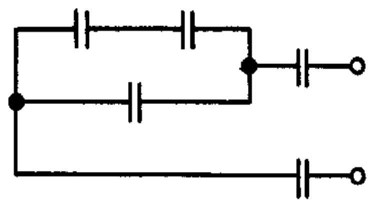
C_T = ________

c.



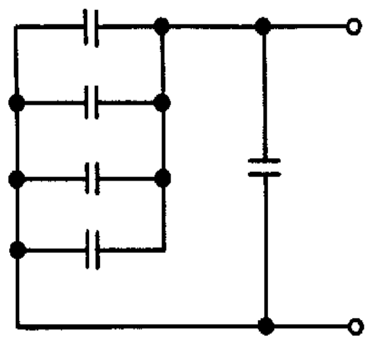
C_T = ________

d.



C_T = ________

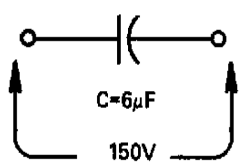
e.



C_T = ________

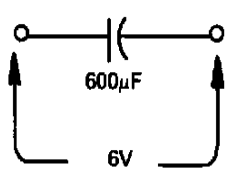
3. Find the following unknown values using the formula Q = CE.

a.



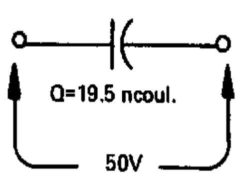
Q = ________

b.



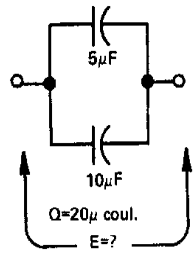
Q = ________

c.



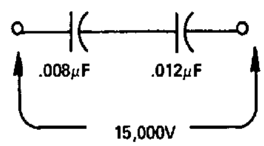
C = ________

d.



E = ________

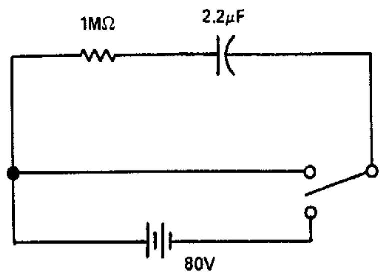
e.



Q = ________

For the circuit shown below, calculate, or use the universal time constant graph to find:

4.



a. RC time constant. ________

b. Time required for capacitor to charge fully. ________

c. Voltage across the capacitor after 1.5 seconds. ________

d. Voltage across the capacitor after 6.5 seconds. ________

e. Time required for the capacitor to charge to 30 volts. ________

Answers

1.a. T = 7.05s

1.b. T = 165 ms

1.c. T = 4.73 ms

1.d. 560 s

1.e. T = 5.6 μs

2.a. C_T = 1 μF

2.b. C_T = 1.33 μF

2.c. C_T = 5 μF

2.d. C_T = 0.75 μF

2.e. C_T = 10 μF

3.a. Q = 900 μC

3.b. Q = 3.6 mC

3.c. C = 390 ρF

3.d. E = 1.33 V

3.e. Q = 72 μC

4.a. T = 2.2s

4.b. 11 s

4.c. 39.5 V

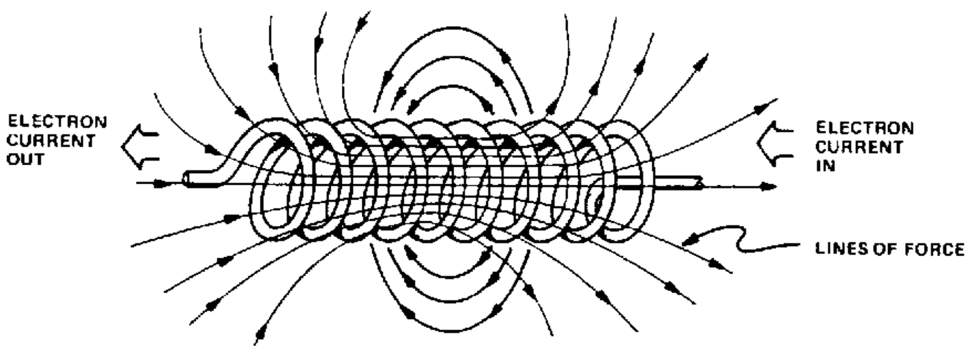
4.d. 75.8 V

4.e. 1.03 s

Inductors and the L/R Time Constant

Worked Through Problems

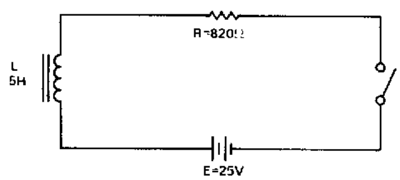
1. Describe the magnetic field around a simple coil of the type shown in the figure below. What is the key effect of a coil's magnetic field on the behavior of coils in dc circuits?

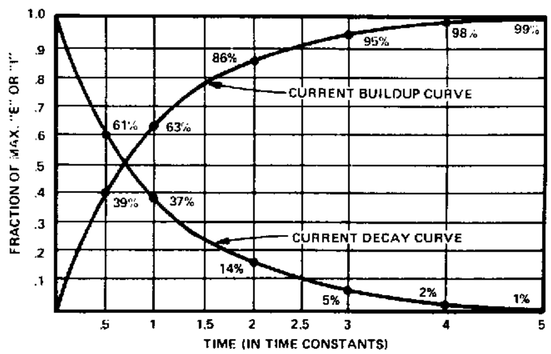


Solution: A magnetic field surrounds any wire carrying current. When this wire is wound into a coil, the magnetic field is concentrated inside the coil as shown by the magnetic lines of force drawn in the figure. This concentrated magnetic field is in effect an energy storage reservoir. Energy is stored when current attempts to increase through the coil, and this energy is released back into the circuit when current attempts to decrease through the coil. For this reason, coils are said to *oppose changes in current* in circuits.

2. Find the following values for the circuit shown below.

   a. Time constant
   b. Maximum steady-state current
   c. Voltage across the resistor after two time constants



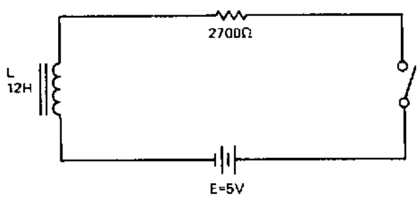


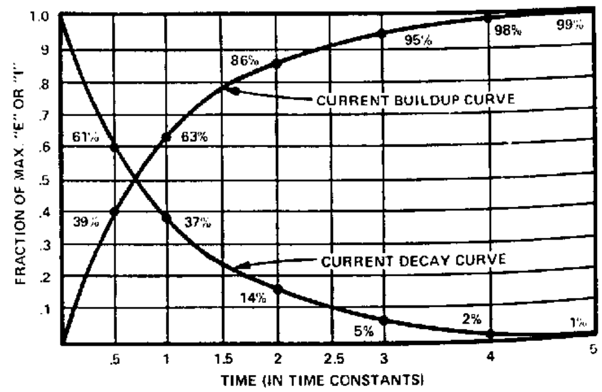
The time constant for this circuit may be found by using the inductive time constant formula, T = L/R. In this circuit, L is equal to 5 henries and R is equal to 820 ohms. 5/820 = 0.0061 second, or 6.1 milliseconds. This is one time constant for this circuit. Five time constants are required for the circuit to reach its steady-state condition. The maximum steady-state current in an inductive circuit is determined by using Ohm's law. The total voltage, E (here 25 volts), must be divided by the total circuit resistance R_T to give you the steady-state current. In this circuit, the total resistance is taken to be 820 ohms, the value of the resistor performing the calculation: 25 V/820 Ω = 30.5 mA. This value of current will be flowing in the circuit after five time constants.

The value of current flowing after only two time constants may be found by using the universal time constant graph. First, locate the two time constant mark on the horizontal line. Trace the graph line up until it intersects the "current buildup" curve. The intersection point is labeled 86%. This means that at this point, the circuit current is at 86% of the steady-state value. So, the current value at 2 time constants may be found by multiplying 0.86 X 30.5 mA. The current flowing after two time constants is equal to 26.2 mA. The value of the current at any time constant point may be determined by using the universal time constant graph in the manner just presented. To find the voltage across the resistor at the end of two time constants, multiply the current at that point (26.2 milliamps), times the resistance (820 ohms), to get your answer (21.5 volts).

3. Find the following values for the circuit shown below:

    a. Time constant
    b. Maximum steady-state current
    c. Voltage across the resistor after 2 milliseconds (2 ms).





Solution:

a. T = L/R

   T = 12/2700

   T = 4.44 ms

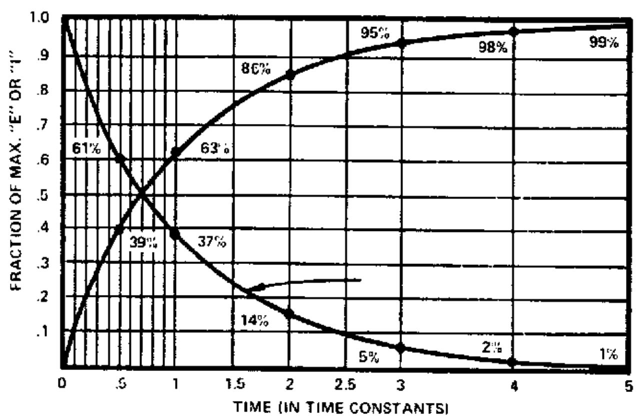
b. $E_T/R_T = I_T$

   5/2700 = 1.85 mA = steady-state current

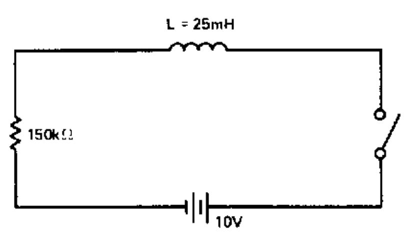
c. To find the circuit current at 2 milliseconds, the first thing to do is locate 2 milliseconds on the horizontal axis of the time constant graph. This axis of the graph is measured out in terms of time constants. You must get the chart to read out in seconds. This may be done by dividing 2 milliseconds by 4.44 milliseconds, to determine the exact percentage 2 milliseconds is as compared to 4.44 milliseconds. Two ms.4.44 ms = 0.45. In terms of time constants, 2 milliseconds is equal to 0.45 (0r 45%) of one time constant. Locate 0.45 on the horizontal axis of the graph. Trace upward until that graph line intersects the current buildup curve. The intersection occurs at approximately 37%. This indicates that the current flowing at this point is 37% of the steady-state current, or 0.37 X 1.85 mA which is equal to 0.68 mA. To find the voltage across the resistor, multiply this current (0.68 milliamps) times the resistance (2700 ohms) to yield the voltage (1.84 volts).

Solve the following problems related to inductance and the L/R time constant, using the time constant formula and the universal time constant graph given below.

Depending upon the approach you use in solving these problems and how you round off intermediate results, your answers may vary slightly from those given here. However, any differences you encounter should only occur in the third significant digit of your answer. If the first two significant digits of your answers do not agree with those given here, recheck your calculations.



1.



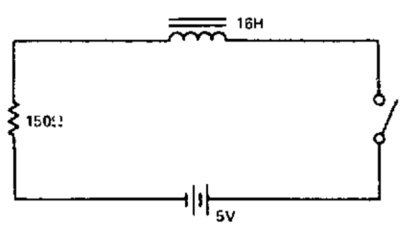
Circuit time constant = ____________

I_{max} = ____________

Voltage across the 150-kilohm resistor

after two time constants = ____________

2.



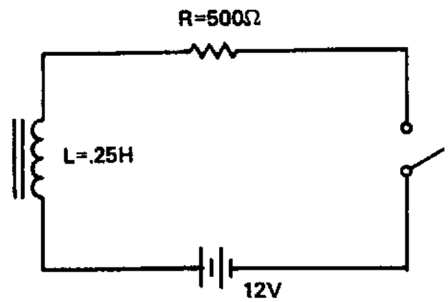
Circuit time constant = ____________

I_{max} = ____________

Voltage across the 150-ohm resistor

after 50 milliseconds = ____________

3.



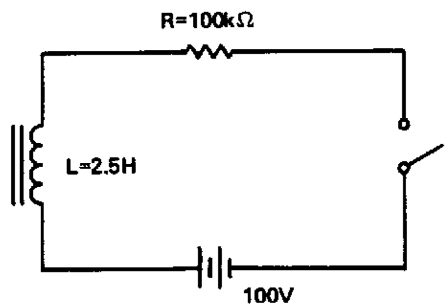
Circuit time constant = ___________

I_{max} = ___________

Voltage across the 500-ohm resistor

after 1 millisecond = ___________

4.



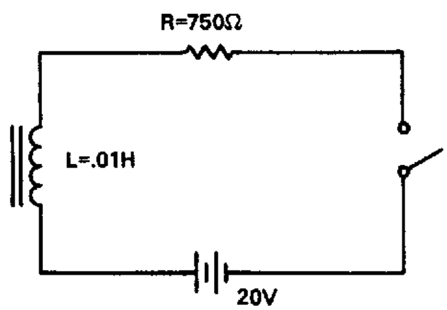
Circuit time constant = ___________

I_{max} = ___________

Voltage across the 100-kilohm resistor

after three time constants = ___________

5.



Circuit time constant = ___________

I_{max} = ___________

Voltage across the 750-ohm resistor

after 25 microseconds = ___________

1. Circuit time constant = 167 nanoseconds

   I_{max} = 66.7 microamps

   Voltage across the 150-kilohm resistor

   after two time constants = 8.6 volts

2. Circuit time constant = 107 milliseconds

   I_{max} = 33.3 milliamps

   Voltage across the 150-ohm resistor

   after 50 milliseconds = 1.86 volts

3. Circuit time constant = 500 microseconds

   I_{max} = 24 milliamps

   Voltage across the 500-ohm resistor

   after 1 milliseconds = 10.3 volts

4. Circuit time constant = 25 microseconds

   I_{max} = 1 milliamp

   Voltage across the 100-kilohm resistor

   after three time constants = 95 volts

5. Circuit time constant = 13.3 microseconds

   I_{max} = 26.7 milliamps

   Voltage across the 750-ohm resistor

   after 25 microseconds = 17 volts

Inductance and Transformers

[redacted]

Transformers

Basic Construction

A device in which the property of mutual inductance is put to practical use is the transformer. A typical transformer is shown in Figure 1. A typical standard transformer consists of two separate coils, wound on a common iron core as shown in the schematic of Figure 2 and considered to have a coefficient of coupling of one. One coil is called the primary; the other is called the secondary. As a result of mutual inductance, a changing voltage across the primary will induce a changing voltage in the secondary. Thus, if the primary winding is connected to an ac source and the secondary to a load resistor, the transformer is able to transfer power from the primary to the secondary to the load resistance as illustrated in Figure 3. By having more or fewer turns in the secondary as compared to the primary, the primary voltage may be either stepped-up or stepped-down to provide the necessary operation voltage for the load.

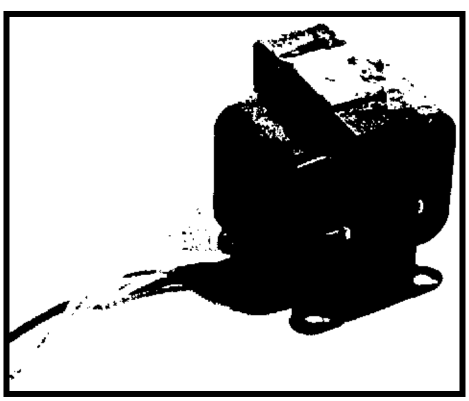


Figure 1
A Typical Transformer

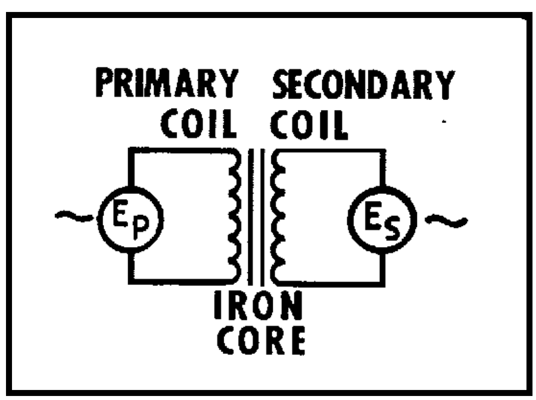


Figure 2
Schematic Drawing of a Transformer

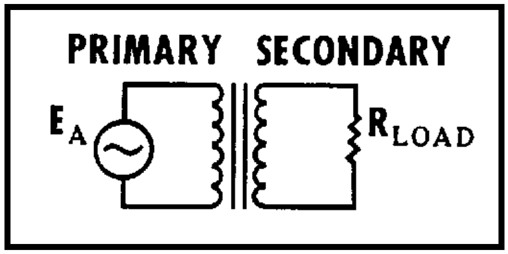
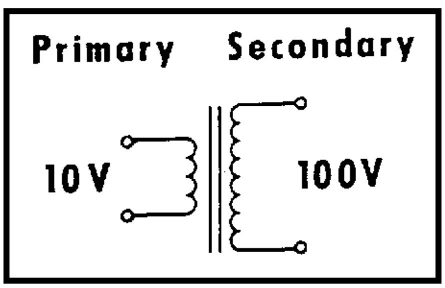


Figure 3

Turns Ratio versus Voltage

Recall that if a coil has a larger number of turns, a larger voltage is induced across the coil. With a smaller number of turns the voltage is less. Therefore it is easy to see that by having more or fewer turns in the secondary as compared to the primary, as shown in Figures 4 and 5, the voltage may either be stepped up or stepped down to provide the necessary operating voltage for the load.



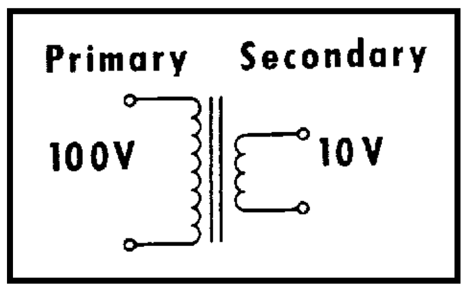


Figure 4
Step-up Transformer

Figure 5
Step-down Transformer

The ratio of the number of turns in a transformer secondary winding to the number of turns in its primary winding is called the turns ratio of a transformer. The equation for turns ratio is:

$$\text{turns ratio} = \frac{N_S}{N_P} \qquad (8\text{-}14)$$

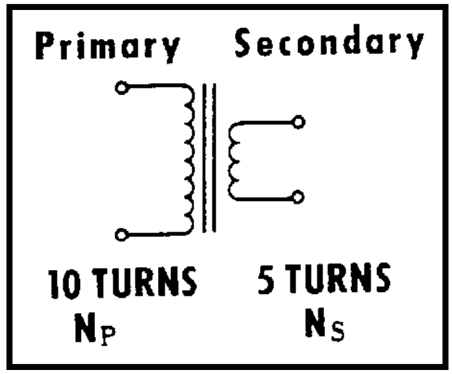


Figure 6
Transformer Used to Calculate Turns Ratio

In the transformer schematic shown in Figure 6, the number of turns in its primary is 10 and the number of secondary turns is 5. Using equation 8-14, the turns ratio of the transformer can be calculated.

$$\text{turns ratio} = \frac{N_S}{N_P} = \frac{5}{10} = \frac{1}{2}$$

Transformers have a unity coefficient of coupling. Therefore, the voltage induced in each turn of the secondary winding (E_{is}) is the same as the voltage self-induced (E_{iP}) in each turn of the primary, as shown in Figure 7. The voltage self-induced in each turn of the primary equals the voltage applied to the primary divided by the number of turns in the primary. This can be written:

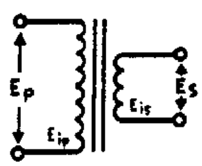


Figure 7
Transformer Voltage Induction

$$E_{iP} = \frac{E_P}{N_P}$$

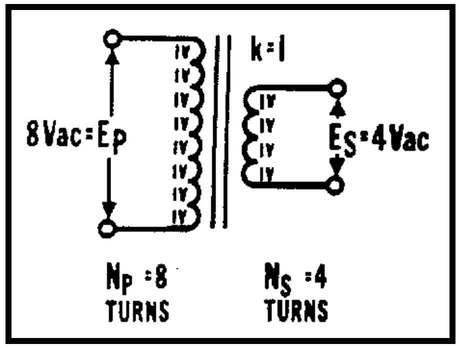


Figure 8
Example Transformer Used to Calculate
Self-Induced Voltage in Primary Turns

Figure 8 shows a schematic of a transformer in which there are 8 turns in the primary and 8 volts ac is applied to it. Using equation 8-15, the voltage self-induced in each primary turn can be calculated.

$$\begin{aligned} E_{iP} &= \frac{E_P}{N_P} \\ &= \frac{8}{8} \\ &= 1V \end{aligned}$$

In this example, one volt is induced in each turn of the primary.

If each turn of the secondary has the same voltage induced in it, then the secondary voltage is equal to the number of secondary turns times the induced voltage. This can be written

$$E_S = N_S\left(\frac{E_P}{N_P}\right) \quad (8\text{-}16)$$

Or rearranging,

$$E_S = E_P\left(\frac{N_S}{N_P}\right) \quad (8\text{-}17)$$

The transformer shown in Figure 8 has 4 turns in its secondary. Using equation 8-16, the secondary voltage can be calculated.

$$
\begin{aligned}
E_S &= N_S\left(\frac{E_P}{N_P}\right) \\
&= 4\left(\frac{8}{8}\right) \\
&= 4V
\end{aligned}
$$

The transformer's secondary voltage is 4 volts—4 turns times 1 volt per turn.

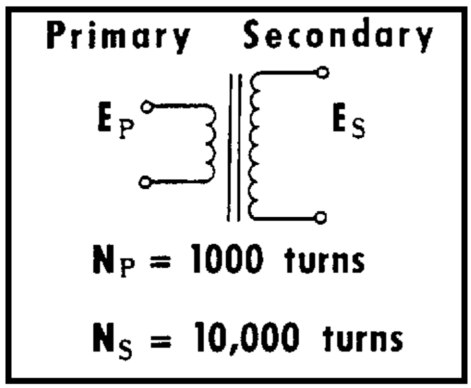


Figure 9
Example for Calculating Turns Ration and E_S

In another example, shown in Figure 9, there are 1000 turns in the primary winding of the transformer and there are 10,000 turns in its secondary winding. Thus, the turns ratio is

$$
\begin{aligned}
\text{turns ratio} &= \frac{N_S}{N_P} \\
&= \frac{10{,}000}{1{,}000} \\
&= \frac{10}{1} \\
&= 10
\end{aligned}
$$

Therefore, the secondary voltage would always be 10 times greater than the primary voltage. If the primary voltage is 10 volts ac, then the secondary voltage will be

$$\begin{aligned} E_S &= 10E_P \\ &= 10(10V) \\ &= 100V \end{aligned}$$

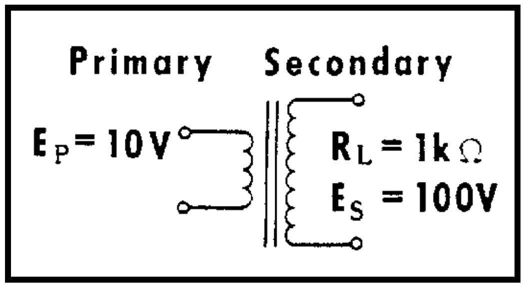


Figure 10
Example for Calculating Transformer I_S

Transformer secondary current is a function of secondary voltage and load resistance. If a 1 kilohm load is placed across the secondary as shown in Figure 10, then the secondary current, by Ohm's law, will be

$$\begin{aligned} I_S &= \frac{E_S}{R_L} \\ &= \frac{100V}{1k\Omega} \\ &= 0.1A \\ &= 100mA \end{aligned}$$

The secondary current is 100 mA. The transformer secondary acts as an ac voltage source to the load.

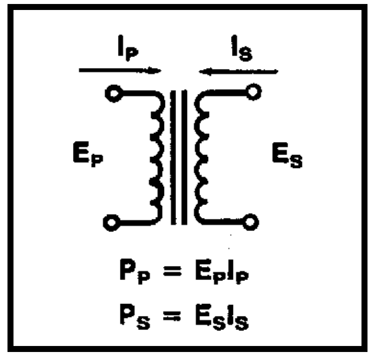


Figure 11
Relationship of Transformer
Primary and Secondary Windings

Primary-to-Secondary Current Relationship

Modern transformers, with coefficient of coupling considered to be one, and with no real power consumed in the windings or the core can be considered to have no loss, as shown in Figure 11. Therefore, the power in the primary is considered to be the same as the power in the secondary, $P_P = P_S$. Since $P = EI$,

$$P_P = P_S$$
$$E_P I_P = E_S I_S$$

Rewriting this,

$$\frac{I_P}{I_S} = \frac{E_S}{E_P} \quad (8\text{-}18)$$

Note that the current relationship is the inverse of the voltage relationship. Thus, if the voltage is stepped up in a transformer by a factor of 10, the current must have been stepped down the same factor. This may be stated another way using equations 8-17 and 8-18.

Since $$\frac{I_P}{I_S} = \frac{E_S}{E_P} = \frac{N_S}{N_S}$$

Then $$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Or $$I_P = \left(\frac{N_S}{N_P}\right) I_S$$

Thus, in the example shown in Figure 10, if E_P is 10 volts, E_S is 100 volts, and if I_S, the secondary current, is 100 milliamperes, the primary current, I_P, is calculated as:

$$\begin{aligned}\frac{I_P}{I_S} &= \left(\frac{N_S}{N_P}\right) I_S \\ &= \left(\frac{100}{10}\right) 100\text{mA} \\ &= (10)\ \ 100\text{mA} \\ &= 1000\text{mA} \\ &= 1A\end{aligned}$$

The primary current in the transformer is one ampere.

Performing the following calculations it can be determined that both the primary and secondary power are equal; both are 10 watts.

$$\begin{aligned}P_P &= E_P I_P \\ &= (10\text{V})\,(1\text{A}) \\ &= 10\text{W} \\ P_S &= E_S I_S \\ &= (100\text{v})\,(100\text{mA}) \\ &= 10{,}000\text{mW} \\ &= 10\text{W}\end{aligned}$$

The transformer, then, either steps up or steps down the voltage and current, but conserves power from the primary to the secondary.

However, transformers do not affect the frequency of the ac voltage they act upon. If the frequency of the primary voltage and current is 60 hertz, then the secondary voltage and current will have a 60 hertz frequency.

Recall that a transformer will not operate with a dc voltage. That is because dc voltage is non-changing and cannot produce an expanding or collapsing magnetic field to cut the secondary windings to produce a secondary voltage.

Variable-output Transformers

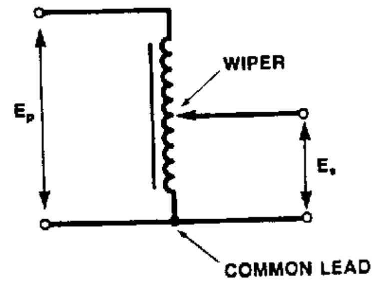


Figure 12
Variable-Output Autotransformer

Some manufactures produce a type of autotransformer that has a variable output voltage. As shown in Figure 12, this is accomplished by making the secondary tap a wiper-type of contact (much like a wire-wound variable resistor). By varying the position of the wiper contact, various output voltages are obtainable. Of course, the same effect could also be produced using a variable tap on the secondary of a two-winding transformer as shown in Figure 13.

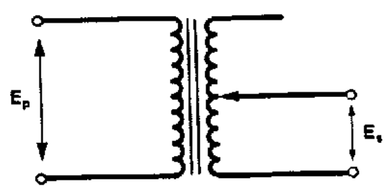


Figure 13
Variable-Output Transformer

Multiple-secondary Transformers

Transformers are also produced which have multiple-secondary and center-tapped secondary windings in order to provide for circuits requiring several different voltage levels. A schematic for a typical multiple-secondary transformer is shown in Figure 14.

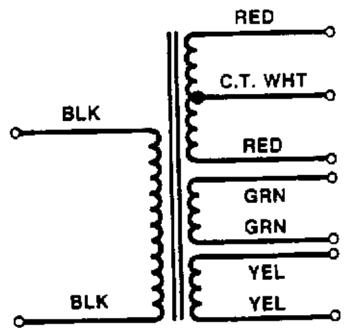


Figure 14
Multiple-Secondary Power Transformer

Transformer Lead Color Code

Transformer leads are usually color coded using a standardized EIA wire color coding technique. A chart showing the standard EIA color code is provided in the appendix. Not all manufacturers use this particular color code so there will be some variation.

Transformer Specifications

Manufacturers provide specifications for transformers. The specifications enable a user to select a transformer that best meets the requirement of the application. Transformer specifications usually include primary voltage and frequency, secondary voltage(s), impedance, dc winding resistance, and current capabilities. For example, the power transformer of Figure 14 has the following specifications:

Primary voltage: 117V, 60 Hz

High-voltage secondary: 240V-0-240V (center-tapped) 150 mA

Low-voltage secondary: 6.3V, 2A

Low-voltage secondary: 5V, 3A

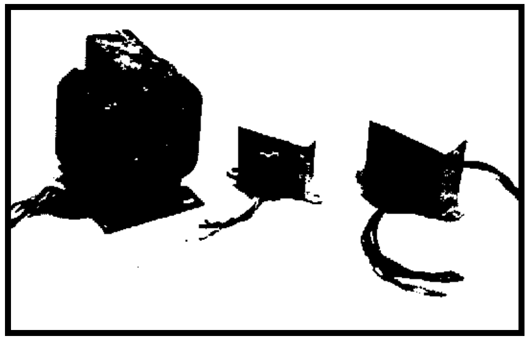


Figure 15
Typical Power, Audio, and
Filament Transformers

Power transformers are multiple secondary winding transformers with both high and low voltage secondaries. Typical power, audio, and filament transformers are shown in Figure 15. Power transformers originally were developed for use with vacuum tube circuits in which high voltage for power supply levels and low voltage for vacuum tube filaments (heaters) were needed. The primary ratings specify the voltage and frequency at which the transformer is designed to be operated. The secondary ratings specify the voltages available from the various secondary windings as well as the maximum current which the secondaries can supply.

Audio transformers are designed for input/output audio applications and are rated according to their primary and secondary impedances, power capabilities (wattage), and turns ratio. They have only a single secondary winding.

Filament transformers are single secondary low voltage, high current (several amperes, typically) transformers rated according to their primary voltage and frequency, secondary output voltage and maximum output current capabilities.

[redacted]

Inductive Reactance

Now that inductance, self- inductance, and transformer action have been discussed, the next step is a discussion of the effect of an inductor in an ac circuit.

Inductance is measured and inductors are rated in henrys. An inductor's effect in a circuit depends on the inductance and is expressed in a quantity called inductive reactance. Inductive reactance is a quantity that represents the opposition that a given inductance presents to an ac current in a circuit, such as is shown in Figure 16.

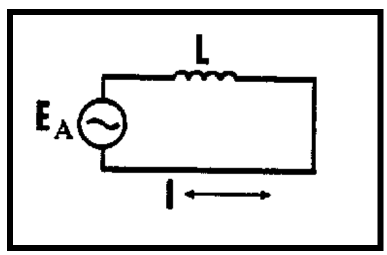


Figure 16
Simple Inductive Circuit

Like capacitive reactance, it is measured in ohms and depends upon the frequency of the applied ac voltage and the value of the inductor. Inductive reactance can be expressed as follows:

$$X_L = 2\pi f L \quad (8\text{-}21)$$

Where

X_L = inductive reactance (ohms
$2\pi = 6.28$
f = frequency (Hz)
L = inductance (H)

The constant of 2π comes from the number of radians in one cycle of a sinusoidal ac waveform. Because of this, this equation is valid only for calculating the inductive reactance of an inductor with sinusoidal alternating current applied.

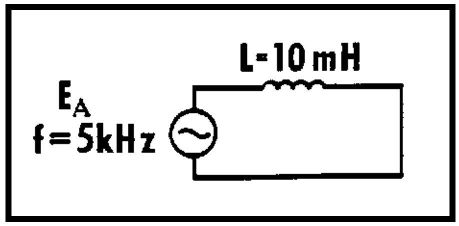


Figure 17
Example Circuit for Calculating Inductive Reactance

Figure 17 shows a simple inductive circuit. The inductor's value is 10 millinery. Applied frequency is 5 kilohertz. Using equation 8-21, inductive reactance, X_L, is calculated:

$$
\begin{aligned}
X_L &= 2\pi fL \\
&= (6.28)(5 \text{ x } 10^3 \text{ Hz})(10 \text{ x } 10^{-3} \text{ H}) \\
&= 314\Omega
\end{aligned}
$$

Note from equation 8-21 that if either the frequency or the inductance is increased the inductive reactance increases. Figure 18 shows graphically how a change in either the frequency or inductance changes the inductive reactance, X_L. Note that the inductive reactance increases linearly with frequency and inductance. As the frequency or inductance increases, the inductor's opposition to the flow of current increases.

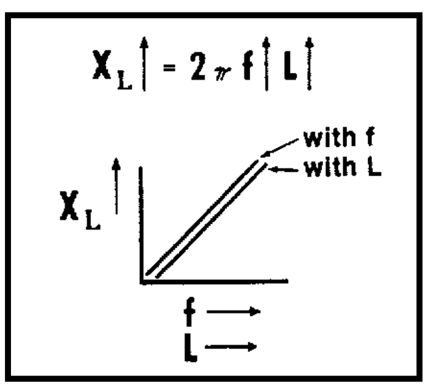


Figure 18
Frequency and Inductance
Versus Inductive Reactance

These plots of inductive reactance versus frequency and inductive reactance versus inductance shown in Figures 19 and 20 will be examined more closely to help you understand these relationships more clearly.

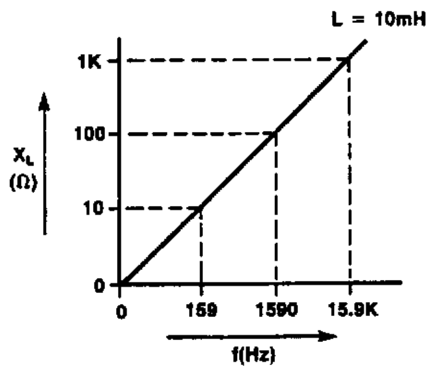


Figure 19
Inductive Reactance Versus
Frequency for an
Inductance of 10 mH

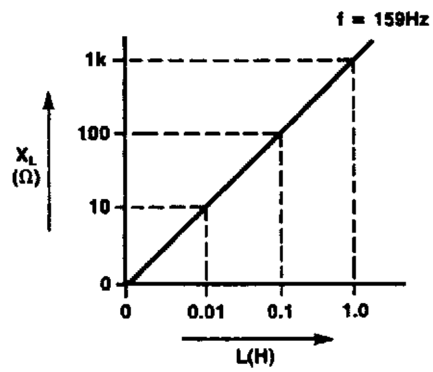


Figure 20
Inductive Reactance Versus
Inductance at a
Frequency of 159 Hz

Figure 19 shows the inductive reactance versus frequency for an inductance of 10 millihenrys. It can be seen that as frequency increases so does the inductive reactance. For example, at a frequency of 159 hertz, the inductive reactance is 10 ohms. However, at a frequency of 1590 hertz the inductive reactance is now 100 ohms. Inductive reactance is directly proportional to frequency.

In Figure 20, which plots inductive reactance versus inductance at a frequency of 159 hertz, it can be seen that as inductance increases so does the inductive reactance. For example, with an inductance of 0.01 henrys (10 millihenrys), inductive reactance is 10 ohms. However, if the inductance is increased to 1 henry, the inductive reactance is now 1 kilohm. Inductive reactance also is directly proportional to inductance.

The basic equation for inductive reactance may be rewritten in two other forms:

$$f = \frac{X_L}{2\pi L} \qquad (8\text{-}22)$$

Or

$$L = \frac{X_L}{2\pi f} \qquad (8\text{-}23)$$

Equation 8-22 can be used to determine the frequency at which an inductance will produce a certain reactance. Equation 8-23 can be used to determine the inductance that will have a certain reactance at a certain frequency. For example, equation 8-22 can be used to determine the frequency at which an 8.5 henry inductor will have an inductive reactance of 5 kilohms.

$$\begin{aligned} f &= \frac{X_L}{2\pi L} \\ &= \frac{5000\Omega}{(628)(8.5H)} \\ &= 93.7\text{Hz} \end{aligned}$$

Equation 8-23 can be used to determine the value of inductance needed to produce an inductive reactance of 10 kilohms at a frequency of 300 kilohertz.

$$\begin{aligned} f &= \frac{X_L}{2\pi f} \\ &= \frac{10\text{x}10^3\,\Omega}{(6.28)(300\text{ x}10^3\text{ Hz})} \\ &= 0.531\text{ x }10^{-2}\text{ H} \\ &= 5.31\text{mH} \end{aligned}$$

Similar calculations can be performed to obtain the reactive power for the parallel inductive circuit. Recall in that circuit I_{L1} = 15.9 milliamperes and I_{L2} = 31.8 milliamperes. Remember the voltage across each branch is the applied voltage. The reactive power of L_1 is:

$$\begin{aligned} P_{XT} &= E_{L1} + I_{L1} \\ &= (40V)\,(15.9mA) \\ &= 636mVAR \end{aligned}$$

The reactive power of L_2 is:

$$\begin{aligned} P_{L2} &= E_{L2} + I_{L2} \\ &= (40V)\,(31.8mA) \\ &= 1272mVAR \end{aligned}$$

The total reactive power is:

$$\begin{aligned} P_{XT} &= P_{L1} + P_{L2} \\ &= 636mVAR + 1272mVAR \\ &= 1908mVAR \end{aligned}$$

Also, the total reactive power in a parallel circuit equals the total applied voltage times the total current.

$$\begin{aligned} P_{XT} &= E_{A} + I_{T} \\ &= (40V)\,(47.7mA) \\ &= 1908mVAR \end{aligned}$$

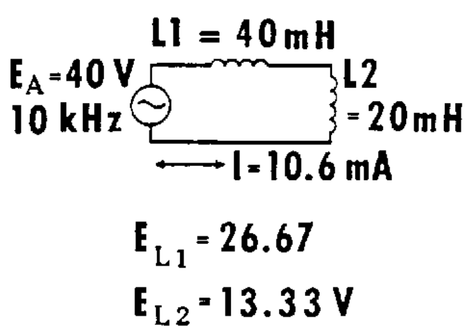


Figure 21
Example Series Inductive Circuit

Summary

This lesson has been an introduction to the inductor, how it is structured, its schematic symbol, its typical units of inductance, and how it functions in typical circuits. The phase relationship of the voltage and current in an inductive circuit were discussed. Mutual inductance and how it is put to use in transformers, and how to make voltage and current calculations for transformer circuits were also discussed. Services and parallel inductive problems were solved, and reactive power calculations were described.

Worked-Out Examples

1. Describe the action of an inductor in a circuit.

Solution:: A magnetic field surrounds any wire carrying current. As current increases through a wire, the magnetic field expands through the wire inducing a counter current which opposes the increase in the initial current. As current decreases in a wire, the magnetic field collapses through the wire inducing current in the same direction and aiding the current which is trying to decrease, thus opposing the decrease of current. when the wire is wound into a coil, the magnetic field produced by each turn of wire in the coil interacts with adjacent turns increasing this inductive effect. This coil of wire is called an inductor. If it is placed in a circuit such that a changing current passes through it, it will oppose the change (increase or decrease) of current.

2. Define inductance.

Solution: Inductance is the property of a circuit which opposes any change in current.

3. If the current through a 8 millihenry-coil is changing at the rate of 10 milliamperes every 5 seconds, determine the rate of change of the current in amperes per second, and the voltage (CEMF) induced across the coil.

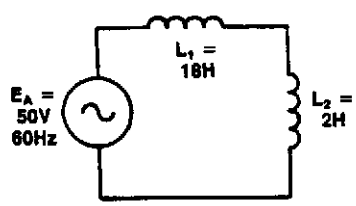
a. Rate of change of current$= \frac{\Delta i}{\Delta t} = \frac{10mA}{5\,sec} = 2mA/sec$

b. CEMF=E_L

$$= L\left(\frac{\Delta i}{\Delta t}\right) = (8mH)\ (2mA/sec) = (8 \times 10^{-3} H)\ (2 \times 10^{-3} A/sec)$$

$$= 16 \times 10^{-6} V = 16 \mu V$$

4. If two coils are connected in series as shown, determine their total inductance with no mutual inductance, and their mutual inductance and total inductance considering mutual inductance (aiding and opposing) if k=0.4.



Solution::

a. L_T (no L_M) = $L_1 + L_2$ = 18H + 2H = **20H**

b. L_T (aid) = $L_1 + L_2 + 2L_M$ = 18H + 2H + 2(2.4H)

= 18H + 2H + 4.8H = **24.8H**

where $L_M = k \cdot \sqrt{L1 \times L2} = 0.4 \cdot \sqrt{18H \times 2H}$

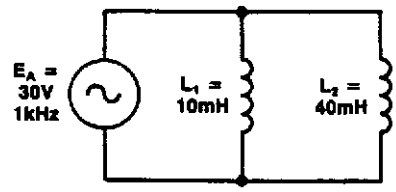
$= 0.4 \cdot \sqrt{36H} = 0.4\ (6)H = \mathbf{2.4H}$

c. L_T (oppose) = $L_1 + L_2 - 2L_M$ = 18H + 2H - 2(2.4H)

L_T (oppose) = 20H - 4.8H = **15.2H**

5.

a. Given the circuit shown solve for the total inductance of the parallel-connected inductors if there is no mutual inductance.



Solution::

$$L_T = \frac{L_1 \text{ x } L_2}{L_1 \text{ x } L_2} = \frac{(10\text{mH})(40\text{mH})}{10\text{mH} + 40\text{mH}} = \left(\frac{400}{50}\right)\text{mH}$$

$$L_T = \mathbf{8mH}$$

b. Determine their mutual inductance and total inductance (aiding and opposing) if mutual inductance exists with a coefficient of 0.2.

Solution:

$$L_M = k\sqrt{L_1 \text{ x } L_2} = 0.2\sqrt{10\text{mH x } 40\text{mH}} = 0.2\sqrt{400mh} = 0.2(20)mH$$

$$L_M = \mathbf{4mH}$$

$$L_T(\text{aid}) = \frac{(L_1 + L_M)(L_2 + L_M)}{L_1 + L_2 + 2L_M} = \frac{(10\text{mH} + 4\text{mH})\ (40\text{mH})}{10\text{mH} + 40\text{mH} + 2(4\text{mH})} = \frac{(14\text{mH})(44\text{mH})}{(58\text{mH})}$$

$$L_T(\text{aid}) = \mathbf{10}.62mH$$

$$L_T(\text{oppose}) = \frac{(L_1 + L_M)(L_2 + L_M)}{L_1 + L_2 + 2L_M} = \frac{(10\text{mH} - 4\text{mH})\ (40\text{mH-}4\text{mH})}{10\text{mH} + 40\text{mH} - 2(4\text{mH})} = \mathbf{5.14mH}$$

6. If the primary voltage applied to a transformer is 120 VAC and the secondary voltage output is 480 VAC, determine the turns ratio for the transformer and state whether it is a step-up or step-down transformer.

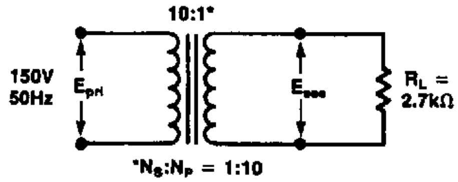
Solution:

a. Turns ratio $\frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{480\text{V}}{120\text{V}} = \frac{4}{1}$

or written in NS:NP form, **4:1**

b. **This is a step-up transformer since the secondary voltage is higher than the primary voltage**.

7. Given the transformer with turns-ratio and load-resistance specified, determine the following values: E_{sec}, I_{sec}, I_{pri}, P_{pri} and P_{sec}. (Assume 100 percent efficiency.)



Solution:

$$E_{sec} = \left(\frac{N_S}{N_P}\right) E_{pri} = \left(\frac{1}{10}\right)\ 150\text{V} = 15\text{V (This is also 50 hertz}$$

$$I_{sec} = \frac{E_{sec}}{R_L} = \frac{15\text{V}}{2.7\text{k}\Omega} = 5.56\text{mA}$$

$$I_{pri} = \left(\frac{N_S}{N_P}\right) I_{sec} = \left(\frac{1}{10}\right)\ 5.56\text{mA} = 0.556\text{mA}$$

$$P_{pri} = E_{pri} \text{ x } I_{pri} = (150\text{V})\ (0.556\text{mA}) = 83.4\text{mW}$$

$$P_{sec} = E_{sec} \text{ x } I_{sec} = (15\text{V})\ (0.56\text{mA}) = 83.4\text{mW}$$

Note that $P_{pri} = P_{sec}$!

8. If the primary voltage is 120 VAC with a primary current of 10 mA and the secondary voltage is 12.6 VAC with a secondary current of 85 millamperes, determine the percent efficiency of this transformer. Explain the loss of power between primary and secondary.

Solution:

a. $P_{pri} = E_{pri} \text{x} I_{pri} = (120\text{V})\ (10\text{mA})\ \ 120\text{mW}$

$$Ps_{ec} = E_{sec} \text{ x } I_{sec} = (12.6\text{V})\,(85\text{mA}) = 1071\text{mW}$$

$$\%\text{Eff} = \frac{P_S}{P_P} \text{ x } 100\% = \left(\frac{1071\text{mW}}{1200\text{mW}}\right) \text{ x } 100\%\ 0.893 \text{ x } 100\% = 89.3\%$$

b. The power loss (10.7 percent of the primary power) Between primary and secondary is due to eddy currents, hysteresis and winding resistance heat loss.

9. Calculate the inductive reactance of the inductors at these specified frequencies:

a. 10 millihenry coil operated at a frequency of 5 kilohertz:

Solution:

$$X_L = 2\pi fL = (6.28)(5kHz) = (6.28)(5 \text{ x}10^3 \text{ Hz})(10 \text{ x}10^{-3} \text{ H})$$
$$= 314 \text{ x } 10^{\circ}\Omega = 314\Omega$$

b. An 8.5 henry coil operated at a frequency of 60 hertz:

Solution:

$$X_L = 2\pi fL = (6.28)(60Hz)(8.5H) = 3202.8\Omega = 3.2k\Omega$$

c. A 45 microhenry coil operated at a frequency of 1250 kilohertz:

Solution:

$$X_L = 2pfL = (6.28)(1250kHz)(45\mu H) = (6.28)(1250 \text{ x } 10^3 \text{ Hz})(45 \text{ x } 10^{-6} \text{ H})$$
$$= 353250 \text{ x}10^{-3}\Omega = 353.25\Omega$$

10. Calculate the value of the inductor needed to produce the reactance specified at the given frequency:

a. A reactance of 1megohm at a frequency of 40 kilohertz:

Solution:

$$L = \frac{XL}{2pf} = \frac{1M\Omega}{(628)(2240kHz)} = \frac{1 \text{ x } 10^6\Omega}{(6.28)(40 \text{ x } 10^3 \text{ Hz})} = 3.9H$$

b. A reactance of 47 kilohms at a frequency of 108 megahertz:

Solution:

$$L = \frac{XL}{2\pi f} = \frac{47k\Omega}{(6.28)(1080MHz)} = \frac{1 \text{ x } 10^6\ \Omega}{(6.28)(108 \text{ x } 10^6 \text{ Hz})} = 3.9H$$

$$= 0.0693 \text{ x } 10^{-3} \text{ H} = 0.0693mH = 69.3\mu H$$

11. Calculate the frequency at which the given inductors will have the specified reactance.

a. A reactance of 50 kilohms with a 4 millihenry inductor:

Solution:

$$f = \frac{X_L}{2\pi f} = \frac{50k\Omega}{(6.28)(4mHz)} = \frac{50 \text{ x } 10^3 \Omega}{(6.28)(5 \text{ x } 10^{-3} H)} = \frac{50 \ x \ 10^3}{25.12 \ x \ 10^{-3}}$$

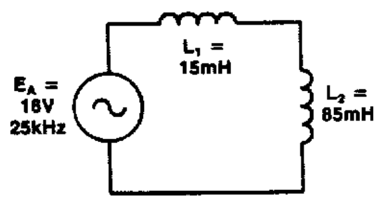
$$= 2 \text{ x } 10^6 \text{ Hz} = 2MHz$$

b. A reactance of 25 ohms with a 5 millihenry inductor:

Solution:

$$f = \frac{X_L}{2\pi f} = \frac{25k\Omega}{(6.28)(5mHz)} = \frac{25\Omega}{(6.28)(5 \text{ x } 10^{-3} H)} = \frac{25}{0.0314} = 796Hz$$

12. Solve for the values indicated using the circuit shown. (Assume L_M = 0.)



a. L_T =	____________	f. E_{L1} =	____________
b. X_{L1} =	____________	g. E_{L2} =	____________
c. X_{L2} =	____________	h. P_{LI} =	____________
d. X_{LT} =	____________	i. P_{L2} =	____________
e. I_T =	____________	j. P_{LT} =	____________

Solution:

a. $L_T = L_1 + L_2 = 15mH + 85mH = 100mH$

b. $X_{L1} = 2\pi fL_1 = (6.28)(25kHz)(15mH) = 2355\Omega = 2.36k\Omega$

c. $X_{L2} = 2\pi fL_2 = (6.28)(25kHz)(85mH) = 1345\Omega = 1.35k\Omega$

d. $X_{LT} = X_{L1} + X_{L2} = 2.36k\Omega + 13.35k\Omega = 15.7k\Omega$ or

$X_{LT} = 2\pi fL_T = (6.28)(25kHz)(100mH) = 1345\Omega = 15.7k\Omega$

e. $I_T = \dfrac{E_A}{X_{LT}} = \dfrac{16V}{15.7k\Omega} = 1.02mA$

f. $E_{L1} = I_{LI}X_{LI} = I_TX_{LI} = (1.02mA)(2.36k\Omega) = 2.4V$

g. $E_{L2} = I_{L2}X_{L2} = I_TX_{L2} = (1.02mA)(13.35k\Omega) = 13.6V$

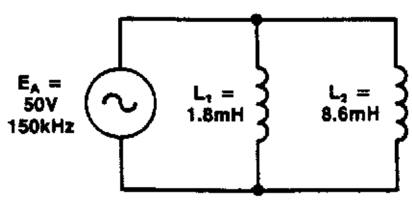
h. $P_{L1} = E_{L1}I_{L1} = E_{L1}I_T = (2.4V)(1.02mA) = 2.45VAR$

i. $P_{L2} = E_{L2}I_{L2} = E_{L2}I_T = (13.6)(1.02mA) = 13.87mVAR$

j. $P_{Lt} = P_{L1}P_{L2} = 2.45mVAr + 13.87mVAR = 16.32mVAR$ or

$P_{LT} = E_AI_T = (16V)(1.02mA) = 16.32mVAR$

13. Solve for the values indicated using the circuit shown. (Assume $L_M = 0$.)



a.	L_T = ____________	f.	I_{L2} = ____________
b.	X_{L1} = ____________	g.	I_T = ____________
c.	X_{L2} = ____________	h.	P_{LI} = ____________
d.	X_{LT} = ____________	i.	P_{L2} = ____________
e.	I_{L1} = ____________	j.	P_{LT} = ____________

Solution:

a. $L_T = \dfrac{L_1}{L_1 + L_2} = \dfrac{(1.8\,\text{mH})\,(8.6\,\text{mH})}{(1.8\,\text{mH} + 8.6\,\text{mH})} = \left(\dfrac{15.48}{10.4}\right)\text{mH} = 1.49\text{mH}$

b. $X_{L1} = 2\pi fL_1 = (6.28)(150\text{kHz})(1.8\text{mH}) = 1695.6\Omega = 1.7\text{k}\Omega$

c. $X_{L2} = 2\pi fL_2 = (6.28)(150\text{kHz})(8.6\text{mH}) = 8101.2\Omega = 8.1\text{k}\Omega$

d. $X_{L2} = \dfrac{(X_{L1})\,(X_{L2})}{X_{LI} + X_{L2}} = \dfrac{(1.7\text{k}\Omega)\,(8.1\text{k}\Omega)}{(1.7\text{k}\Omega) + (8.1\text{k}\Omega)}\left(\dfrac{13.77}{9.8}\right)\text{k}\Omega = 1.4\text{k}\Omega$ or

$X_{LT} = 2\pi fL_T = (6.28)(150\text{kHz})(1.49\text{mH}) = 1403.6\Omega = 1.4\text{k}\Omega$

e. $I_{L1} = \dfrac{E_A}{X_{L1}} = \dfrac{50\text{V}}{1.7\text{k}\Omega} = 29.4\text{mA}$

f. $I_{L2} = \dfrac{E_A}{X_{L2}} = \dfrac{50\text{V}}{8.11\text{k}\Omega} = 6.2\text{mA}$

g. $I_T = I_{L1} + I_{L2} = 20.4\text{mA} + 6.2\text{mA}) = 35.6\text{mA}$

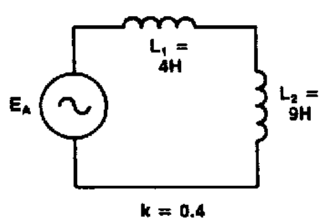
h. $P_{L1} = E_{L1}I_{L1} = E_A I_{L1} = (50\text{V})\,(29.4\text{mA}) = 1470\text{mVAR}$

i. $P_{L2} = E_{L2}I_{L2} = E_A I_{L2} = (50V)\,(6.2\text{mA}) = 310\text{mVAR}$

j. $P_{LT} = E_A I_T = (50\text{V})\,(35.6\text{mA}) = 1780\text{mVAR}$ or

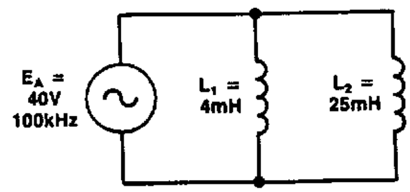
$P_{LT} = P_{LI} + P_{L2} = 1470\text{mVAR} + 310\text{mVAR} = 1780\text{mVAR}$

1. State a short definition of inductance.
2. The concepts of two men are used to explain CEMF for inductors. Who are they?
3. If an iron core is extracted from a coil, will the coil's inductance increase or decrease? Why?
4. As the number of turns of wire used in a coil increases, does the value of its inductance increase or decrease?
5. If two coils are placed in proximity of one another and one coil produces 4000 lines of flux, 3500 of which cut the second coil, what is the coefficient of coupling of these two coils? k= ________.
6. What is the range of values for the coefficient of coupling? ________ to ________. (upper and lower limits for k);
7. In the circuit shown, two coils are connected in series. Determine their total inductance with no mutual inductance. Then determine their mutual inductance and their combined inductance considering mutual inductance (aiding and opposing). k = 0.4, L_1 = 4 henrys. and L_2 = 9 henrys.



   a. L_T(no L_M) = ________.
   b. L_M = ________.
   c. L_T(aid) = ________.
   d. L_T(opp) = ________.

8. In the circuit shown, determine the total inductance of the two parallel-connected inductors if there is no mutual inductance. Then determine their mutual inductance and total inductance (aiding and opposing) if they have a coefficient of coupling of 0.2.



a. L_T(no L_M) = ________.

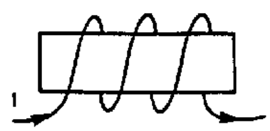
b. L_T(aid) = ________.

c. L_T(opp) = ________.

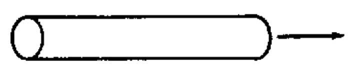
d. L_M = ________.

9. a. Sketch the magnetic field about the coil in the drawing.

Indicate north and south poles.



b. Sketch the magnetic field about the conductor. Show its direction.



10. If the current through a coil is changing at the constant rate of 40 milliamperes every 10 seconds, determine the rate of change of the current in amperes per second. If the coil is rated at 5 millihenrys, determine the voltage across the coil.

a. ROC of I = ______________ A/sec

b. E_L = ______________.

11. What coefficient of coupling is desired for transformers?

k = ______________.

12. If the primary voltage is greater than the secondary voltage of a transformer, is it known as a step-up or step-down transformer?

13. What two types of core losses in a transformer are associated directly with the core?

a. ______________.

b. ______________.

14. If E_P = 120 VAC and E_S = 25.2 VAC, determine the turns ration (N_S:N_P) of the transformer.

Turns ratio = ______________.: ______________.

15. What type of transformer does not provide electrical isolation of primary to secondary?

16. If the primary voltage is 240 VAC with a primary current of 8 milliamperes and the secondary voltage is 50 VAC with a secondary current of 33 milliamperes, determine the percent efficiency of this transformer:

% eff = ______________.

17. A transformer has a turns ration (N_S:N_P) of 1:4.5, has 120 VAC applied to its primary, and has a 6.8 kilohm resistor as a load on its secondary. Determine the secondary voltage, the secondary current, and primary current. (Assume 100 percent efficiency.)

a. E_{sec} = ______________.

b. I_{sec} = ______________.

c. I_{pri} = ______________.

18. When 40 VAC is applied to the primary of a transformer, a secondary current of 8 milliamperes flows through a one kilohm resistor connected across the secondary. 2 milliamperes of

primary current is present. Determine the percent efficiency of the transformer and transformer and its turns ratio.

a. % eff = ______________.

b. N_S:N_P = ______________.

19. Calculate X_L for a 2 millihenry coil operated at frequencies of 100 hertz, 5 kilohertz, and 1.2 megahertz.

a. X_L (f = 100 hertz) = ______________.

b. X_L (f = 5 kilohertz) = ______________.

a. X_L (f = 1.2 megahertz) = ______________.

20. From Problem 19, you see that as the frequency applied to an inductor increases, the inductive reactance of it ______________. (increases, decreases).

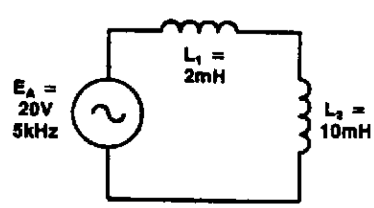
21. What is the value of an inductor needed to produce a reactance of 482 kilohms at a frequency of 5 kilohertz?

L = ______________.

22. What is the frequency at which an inductor of 8.5 henrys will have an inductive reactance of 1 kilohms?

f = ______________.

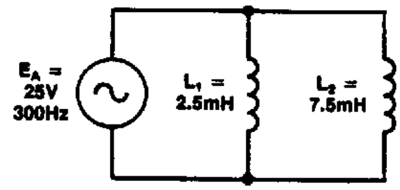
23. Solve for the indicated values using the circuit shown. (Assume L_M = 0.)



a.	X_{L1} = ______________.		f.	E_{L2} = ______________.	
b.	X_{L1} = ______________.		g.	P_{LI} = ______________.	
c.	X_{LT} = ______________.		h.	P_{L2} = ______________.	
d.	I_T = ______________.		i.	P_{L2} = ______________.	

e. E_{L1} = ______________.

24. Solve for the values using the circuit shown. (Assume L_M = 0.)



a.	L_T =	______________.	f.	I_T =	______________.
b.	X_{L1} =	______________.	g.	E_{LI} =	______________.
c.	X_{L2} =	______________.	h.	E_{L2} =	______________.
d.	I_{L1} =	______________.	i.	I_{L2} =	______________.
e.	X_{LT} =	______________.			

1. Inductance is the property of a circuit that

   a. opposes any change in voltage.

   b. opposes current.

   c. opposes any change in current.

   d. opposes any change in frequency.

2. Which of the factors listed below does not govern the value of a coil?

   a. Number of turns

   b. The type of core material used

   c. The size (cross-sectional area) of the coil

   d. The length of the coil

   e. The size of the wire used in the coil

3. The rise or fall of current through an inductor in a circuit is said to be:

   a. exponential

   b. logarithmic

   c. linear

   d. none of the above

4. The voltage that appears across an inductor in a circuit is called______________. and appears only when ______________.the inductor.

   a. counter emf; the current is constant through

   b. voltage drop; the voltage changes across

   c. counter EMF; the current increases or decreases through

   d. voltage drop; the voltage is constant across

5. The phase relationship of the voltage across an inductor and the current passing through it in an ac (sinusoidal) circuit is such that

    a. the voltage lags the current by 90 degrees.

    b. the current leads the voltage by 90 degrees.

    c. the voltage leads the current by 90 degrees.

    d. the voltage and current are in phase.

6. Determine the mutual inductance of two inductors having a coefficient of coupling of 0.8 if their values are 16 millihenrys and 5 millihenrys.

    a. 64 mH

    b. 7.2 mH

    c. 2 0mH

    d. 36 mH

7. If the two inductors are series-connected and their values are 16 henrys and 25 henrys, determine their total inductance if they have no mutual inductance.

    a. 9.76H

    b. 6.4H

    c. 20H

    d. 41H

8. If the two inductors of Question 7 have a coefficient of coupling of 0.2, determine their total inductance aiding and opposing.

    a. 41H, 49H

    b. 49H, 33H

    c. 49H, 41H

    d. 41H, 8H

9. If a transformer has a turns ration of 1:19 (N_S:N_P), an applied primary voltage of 120 VAC, 60 hertz, and a secondary load resistance of 3.3 kilohms, determine the quantities specified below. (Assume 100 percent efficiency.)

   a. E_{sec} =______________________

   b. I_{sec} =______________________

   c. I_{pri} =______________________

   c. I_{pri} = P_{sec} ____________________

10. A transformer has a greater primary current than secondary current under load conditions. Is it a step-up or step-down transformer?

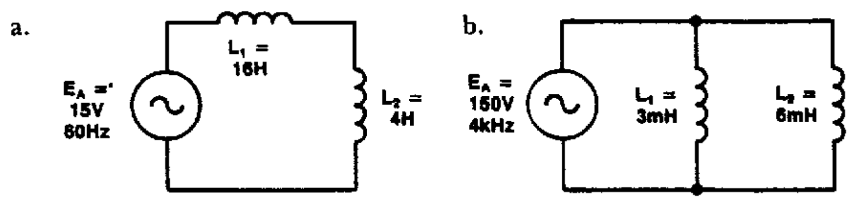
11. Using the inductive reactance equation and given the data specified below, solve for the unknown quantity.

   a. $L = 15\text{mH}, f = 5\text{kHz}: X_L =$ _____________

   b. $X_L = 20\text{k}\Omega, f = 3.5\text{MHz}: L =$ _________

   c. $X_L = 600\text{k}\Omega, L = 10\text{mH}: f =$ _________

12. Determine the requested voltages, currents and power for these two circuits. (Assume L_M = 0.)



Circuit a			Circuit b		
a.	X_{LT} =	______________.	i.	X_{LT} =	______________.
b.	E_{LI} =	______________.	j.	I_{L1} =	______________.
c.	E_{L2} =	______________.	k.	I_{L2} =	______________.
d.	I_T =	______________.	l.	I_T =	______________.
e.	P_{LI} =	______________.	m.	P_{L1} =	______________.
f.	P_{L2} =	______________.	n.	P_{L2} =	______________.
g.	P_{LT} =	______________.	o.	P_{LT} =	______________.

h. L_T = ____________.

1. Inductance is the property of a circuit that opposes any change in current.
2. Oersted and Faraday
3. Decrease. This happens because the permeability of iron is more than that of air and as the iron core is extracted, the permeability of the core is reduced; Thus, the value of the inductance of the coil is decreased.
4. Increase
5. $k = \frac{\phi \text{Common}}{\phi \text{Total}} = \frac{3500}{4000} = 0.875$
6. 0 to 1 (k = 0, no mutual inductance to k = 1, unity coupling)
7. a. 13H

   b. 2.4H

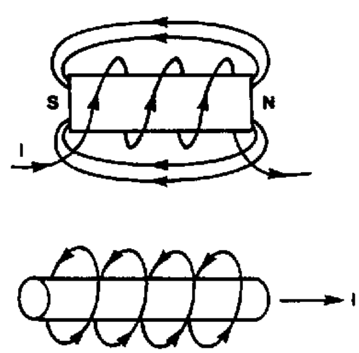
   c. 17.8H

   d. 8.2H
8. a. 3.45 mH

   b. 4.9 mH

   c. 1.84 nG

   d. 2 mH
9.



10. a. $ROC\ of\ i = 4x10^{-3}\ A/sec$

b. $E_L = 20\mu V$

11. $k = 1$

12. Step-down

13. a. hysteresis

b. eddy currents

14. $\frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{25.2V}{120V} = \frac{1}{4.76}$

$N_S : N_P = 1 : 4.76$

15. Autotransformer

16. $P_{pri} = 1920\ mW; P_{sec} = 1650\ mW$

$$\%eff = \frac{P_{sec}}{P_{pri}}\ x\ 100$$

$$= \frac{1650mW}{1920mW} = x\ 100 = 85.9\%$$

17. a. 26.67V

B. 3.92 mA

c. 0.87 mA

18. a. 80%

b. 1:5 (N_S:N_P)

19. a. 1.26Ω

b. 62.8Ω

c. $15k\Omega$

20. Increases

21. 15.4H

22. 18.7 Hz

23. a. 62.8Ω
b. 314Ω
c. 376.8Ω
d. 53mA
e. 3.3V
f. 16.7V
g. 175mVAR
h. 885mVAR
i. 1060mVAR

24. a. 1.875mH
b. 4.71Ω
c. 14.13Ω
d. 5.3A
e. 3.53Ω
f. 7.07A
g. 25V
h. 25V
i. 1.77A

Alternating-Current Circuits

This chapter shows how to analyze sine-wave ac circuits that have R, X_L, and X_C. How do we combine these three types of ohms of opposition, how much current flows, and what is the phase angle? These questions are answered for both series and parallel circuits.

The problems are simplified by the fact that in series circuits X_L is at 90° and X_C at -90°, which are opposite phase angles. Then all of one reactance can be canceled by part of the other reactance, resulting in only a single net reactance. Similarly, in parallel circuits, I_L and I_C have opposite phase angles. These currents oppose each other for one net reactive line current.

Finally, the idea of how ac power and dc power can differ because of ac reactance is explained. Also, types of ac current meters are described including the wattmeter.

Important terms in this chapter are:

apparent power	VAR unit
power factor	voltampere unit
real power	wattmeter

More details are explained in the following sections:

1. AC Circuits with Resistance but no Reactance
2. Circuits with X_L Alone
3. Circuits with X_C Alone
4. Opposite Reactances Cancel
5. Series Reactance and Resistance
6. Parallel Reactance and Resistance
7. Series-Parallel Reactance and Resistance
8. Real Power
9. AC Meters
10. Wattmeters
11. Summary of Types of Ohms in AC Circuits
12. Summary of Types of Phasors in AC Circuits

Combinations of series and parallel resistances are shown in Figure 1. In Figure 1a and b, all voltages and currents throughout the resistive circuit are in the same phase as the applied voltage. There is no reactance to cause a lead or lag in either current or voltage.

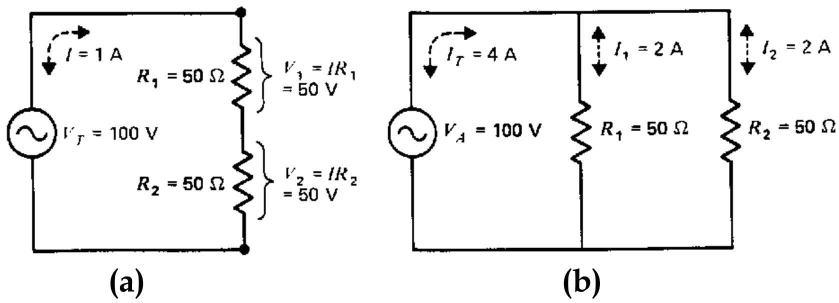


Figure 1
AC Circuits with Resistance but no Reactance
(a) Resistances R_1 and R_2 in series (b) Resistances R_1 and R_2 in Parallel

Series Resistances

For the circuit in Figure 1a, with two 50-Ω resistances in series across the 100-V source, the calculations are as follows:

$$R_T = R_1 + R_2 = 50 + 50 = 100\Omega$$
$$I = \frac{V_T}{R_T} = \frac{100}{100} = 1\text{ A}$$
$$V_1 = IR_1 = 1 \text{ x } 50 = 50\text{ V}$$
$$V_2 = IR_2 = 1 \text{ x } 50 = 50\text{ V}$$

Note that the series resistances R_1 and R_2 serve as a voltage divider, as in dc circuits. Each R has one-half the applied voltage for one-half the total series resistance.

The voltage drops V_1 and V_2 are both in phase with the series current I, which is the common reference. Also I is in phase with the applied voltage V_T because there is no reactance.

Parallel Resistances

For the circuit in Figure 1b, with two 50-Ω resistances in parallel across the 100-V source, the calculations are

$$I_1 = \frac{V_A}{R_1} = \frac{100}{50} = 2\ \text{A}$$

$$I_2 = \frac{V_A}{R_2} = \frac{100}{50} = 2\ \text{A}$$

$$I_T = I_1 + I_2 = 2 + 2 = 4\ \text{V}$$

With a total current of 4 A in the main line from the 100-V source, the combined parallel resistance is 25 Ω. This R_T equals 100 V/4A for the two 50-Ω branches.

Each branch current has the same phase as the applied voltage. Voltage V_A is the reference because it is common to both branches.

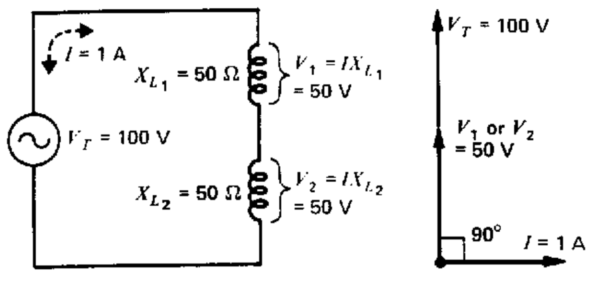
Practice Problems — Section-1

Answers at End of Chapter

a. In Figure 1a, what is the phase angle between V_T and I?

b. In Figure 1b, what is the phase angle between I_T and V_A?

The circuits with X_L in Figures 2 and 3 correspond to the series and parallel circuits in Figure 1, with ohms of X_L equal to the R values. Since the applied voltage is the same, the values of current correspond because ohms of XL are just as effective as ohms of R in limiting the current or producing a voltage drop.



(a) (b)

Figure 2
Series Circuit with X_L Alone
(a) Schematic diagram (b) Phasor diagram

Although X_L is a phasor quantity with a 90° phase angle, all the ohms of opposition are the same kind of reactance in this example. Therefore, without any R or X_C, the series ohms of X_L can be combined directly. Similarly, the parallel I_L currents can be added.

X_L Values in Series

For Figure 2a, the calculations are

$$X_{L_T} = X_{L_1} + X_{L_2} = 50 + 50 = 100\,\Omega$$

$$I = \frac{V_T}{R_{L_T}} = \frac{100}{100} = 1\text{ A}$$

$$V_1 = IX_{L_1} = 1 + 50 = 50\text{ V}$$

$$V_2 = IX_{L_2} = 1 + 50 = 50\text{ V}$$

Note that the two series voltage drops of 50 V each add to equal the total applied voltage of 100 V.

With regard to the phase angle for the inductive reactance, the voltage across any X_L always leads the current through it by 90°. In Figure 2b, "I" is the reference phasor because it is common to all the series components. Therefore, the voltage phasors for V_1 and V_2 across either reactance, or V_T across both reactances, are shown leading "I" by 90°.

I_L Values in Parallel

For Figure 3a the calculations are

$$I_1 = \frac{V_A}{X_{L_1}} = \frac{100}{50} = 2\ \text{A}$$

$$I_2 = \frac{V_A}{X_{L_2}} = \frac{100}{50} = 2\ \text{A}$$

$$I_T = I_1 + I_2 = 2 + 2 = 4\ \text{A}$$

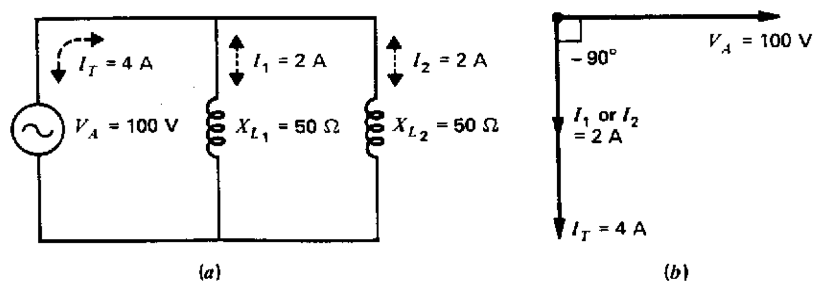


Figure 3
Parallel Circuit with X_L Alone
(a) Schematic Diagram (b) Phasor Diagram

These two branch currents can be added because they both have the same phase. The angle is 90° lagging the voltage reference phasor as shown in Figure 3b.

Since the voltage V_A is common to the branches, this voltage is across X_{L_1}, and X_{L_2}. Therefore V_A is the reference phasor for parallel circuits.

Note that there is no fundamental change between Figure 2b, which shows each X_L voltage leading its current by 90°, and Figure 3b, showing each X_L current lagging its voltage by -90°. The phase angle between the inductive current and voltage is still the same 90°.

a. In Figure 2, what is the phase angle of V_T with respect to I?

b. In Figure 3, what is the phase angle of I_T with respect to V_A?

Circuits With X_C Alone

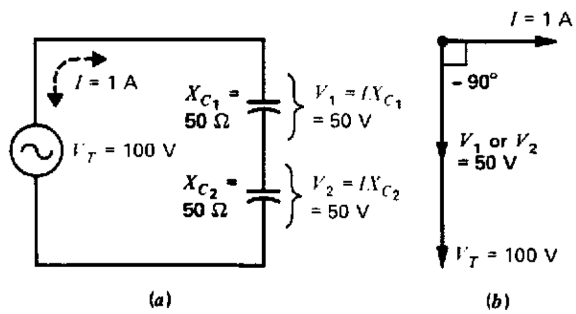


Figure 4
Series Circuit With X_C Alone
(a) Schematic Diagram (b) Phasor Diagram

Again, reactances are shown in Figures 4 and 5 but with X_C values of 50 Ω. Since there is no R or X_L, the series ohms of X_C can be combined directly. Also the parallel I_C currents can be added.

X_C Values in Series

For Figure 4a, the calculations for V_1 and V_2 are the same as before. These two series voltage drops of 50 V each add to equal to total applied voltage.

With regard to the phase angle for the capacitive reactance, the voltage across any X_C always lags its capacitive charge and discharge current "I" by 90°. For the series circuit in Figure 4, "I" is the reference phasor. The capacitive current leads by 90°. Or, we can say that each voltage lags "I" by -90°.

I_C Values in Parallel

For Figure 5, V_A is the reference phasor. The calculations for I_1 and I_2 are the same as before. However, now each of the capacitive branch currents or the I_T leads V_A by 90°.

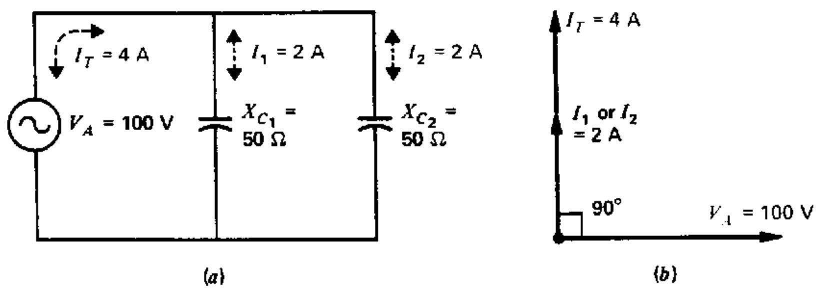


Figure 5
Parallel Circuit With XC Alone
(a) Schematic Diagram (b) Phasor Diagram

Practice Problems — Section 3

a. In Figure 4, what is the phase angle of V_T with respect to I?

b. In Figure 5, what is the phase angle of I_T with respect to V_A?

Opposite Reactances Cancel

In a circuit with both X_L and X_C, the opposite phase angles enable one to cancel the effect of the other. For X_L and X_C in series, the net reactance is the difference between the two series reactances, resulting in less reactance than either one. In parallel circuits, the I_L and I_C branch currents cancel. The net line current then is the difference between the two branch currents, resulting in less total line current than either branch current.

X_L and X_C in Series

For the example in Figure 6, the series combination of a 60-Ω X_L and a 40-Ω X_C in Figure 6a and b is equivalent to the net reactance of the 20-Ω X_L shown in Figure 6c. Then, with 20 Ω as the net reactance across the 120-V source, the current is 6 A. This current lags the applied voltage V_T by 90° because the net reactance is inductive.

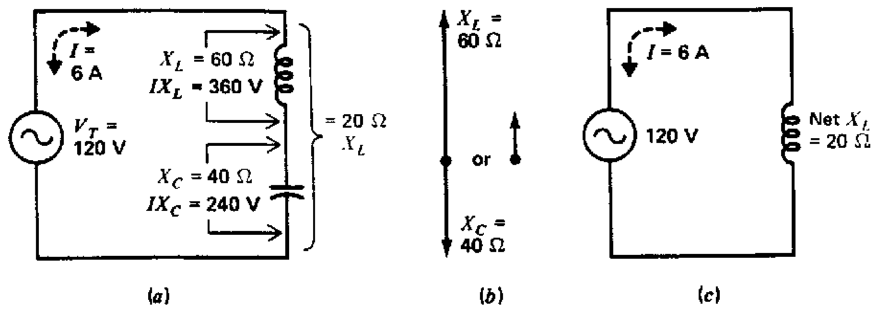


Figure 6
When XL and XC Are in Series, Their Ohms of Reactance Cancel
(a) Series Circuit (b) Phasors for X_L and X_C With Net Resultant
(c) Equivalent Circuit with Net Reactance of 20 Ω of X_L

For the two series reactances in Figure 6a, the current is the same through both X_L and X_C. Therefore, the voltage drops can be calculated as

$$V_L \text{ or } IX_L = 6 \text{ A} \times 60\ \Omega = 360 \text{ V}$$

$$V_C \text{ or } IX_C = 6 \text{ A} \times 40\ \Omega = 240 \text{ V}$$

Note that each individual reactive voltage drop can be more than the applied voltage. The sum of the series voltage drops still is 120 V, however, equal to the applied voltage. This results because the IX_L and IX_C voltages are opposite. The IX_L voltage leads the series current by 90°; the IX_C voltage lags the same current by 90°. Therefore, IX_L and IX_C are 180° out of phase with each other, which means they are of opposite polarity and cancel. Then the total voltage across the two in series is 360 V minus 240 V, which equals the applied voltage of 120 V.

If the values in Figure 6 were reversed, with an X_C of 60 Ω and an X_L of 40 Ω, the net reactance would be a 20-Ω X_C. The current would be 6 A again, but with a lagging phase angle of -90° for the capacitive voltage. The IX_C voltage would then be larger at 360 V, with an IX_L value of 240V, but the difference still equals the applied voltage of 120 V.

X_L and X_C in Parallel

In Figure 7, the 60-Ω X_L and 40-Ω X_C are in parallel across the 120-V source. Then the 60-Ω X_L branch current I_L is 2 A, and the 40-Ω X_C branch current I_C is 3 A. The X_C branch has more current because its reactance is less than X_L.

In terms of phase angle, I_L lags the parallel voltage V_A by 90°, while I_C leads the same voltage by 90°. Therefore, the opposite reactive branch currents are 180° out of phase with each other and cancel. The net line current then is the difference between 3 A for I_C and 2 A for I_L, which equals the net value of 1 A. This resultant current leads V_A by 90° because it is capacitive current.

If the values in Figure 7 were reversed, with an X_C of 60 Ω and an X_L of 40 Ω, I_L would be larger. The I_L then equals 3 A, with an I_C of 2 A. The net line current is 1 A again but inductive, with a net X_L.

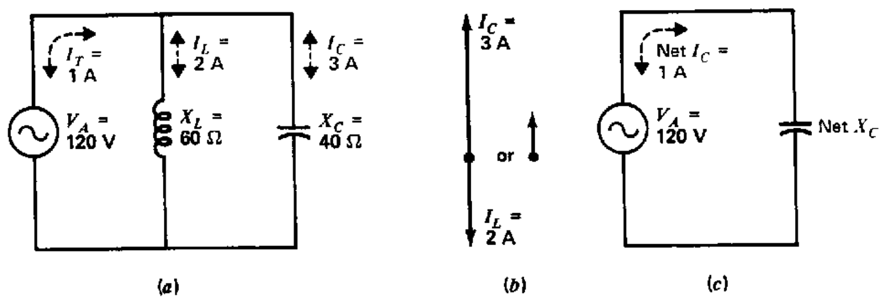


Figure 7
When X_L and X_C are in Parallel, Their Branch Currents Cancel
(a) Parallel Circuit (b) Phasors for Branch Currents I_C and I_L With Net Resultant (c) Equivalent Circuit With Net Line Current of 1 A for I_C

Practice Problems — Section 4

a. In Figure 6, how much is the net X_L?

b. In Figure 7, who much is the net I_C?

Series Reactance and Resistance

In this case, the resistive and reactive effects must be combined by phasors. For series circuits, the ohms of opposition are added to find Z. First added all the series resistances for one total R. Also combine all the series reactances, adding the same kind but subtracting opposites. The result is one net reactance, indicated X.

It may be either capacitive or inductive, depending on which kind of reactance is larger. Then the total R and net X can be added by phasors to find the total ohms of opposition for the entire series circuit.

Magnitude of Z

After the total R and net reactance X are found, they can be combined by the formula

$$Z = \sqrt{R^2 + X^2} \qquad (24\text{-}1)$$

The circuit's total impedance Z is the phasor sum of the series resistance and reactance. Whether the net X is at +90° for X_L or -90° for X_C does not matter in calculating the magnitude of Z.

An example is illustrated in Figure 8. Here the net series reactance in Figure 8b is a 30-Ω X_C. This value is equal to a 60-Ω X_L subtracted from a 90-Ω X_C as shown in Figure 8a. The net 30-Ω X_C in Figure 8b is in series with a 40-Ω R. Therefore

$$\begin{aligned} Z &= \sqrt{R^2 + X^2} \\ &= \sqrt{(40)^2 + (30)^2} \\ &= \sqrt{1600 + 900} = \sqrt{2500} \\ Z &= 50\ \Omega \end{aligned}$$

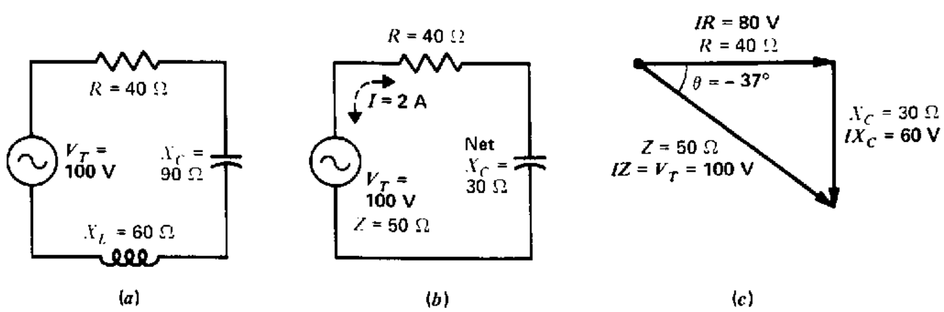


Figure 8
Impedance Z of Series Circuit
(a) Resistance R, X_L, and X_C in Series
(b) Equivalent Circuit With One Net Reactance (c) Phasor Diagram

I = V/Z

The current is 100 V/50 Ω in this example. or 2 A. This value is the magnitude, without considering the phase angle.

Series Voltage Drops

All the series components have the same 2-A current. Therefore, the individual drops in Figure 8a are

$$V_R = IR = 2 \times 40 = 80 \text{ V}$$

$$V_C = IX_C = 2 \times 90 = 180 \text{ V}$$

$$V_L = IX_L = 2 \times 60 = 120 \text{ V}$$

Since IX_C and IX_L are voltages of opposite polarity, the net reactive voltage is 180 minus 120 V, which equals 60 V. The phasor sum of IR at 80 V and the net reactive voltage IX of 60 V equals the applied voltage V_T of 100 V.

Phase Angle of Z

The phase angle of the series circuit is the angle whose tangent equals X/R. The angle is negative for X_C but positive for X_L.

In this example, X is the net reactance of 30 Ω for X_C and R is 40 Ω. Then tan θ = -0.75 and θ is -37°, approximately.

The negative angle for Z indicates lagging capacitive reactance for the series circuit. If the values of X_L and X_C were reversed, the phase angle would by +37°, instead of -37°, because of the net X_L. However, the magnitude of Z would still be the same.

More Series Components

How to combine any number of series resistances and reactances is illustrated by Figure 9. Here the total series R of 40 Ω is the sum of 30 Ω for R_1 and 10 Ω for R_2. Note that the order of connection does not matter, since the current is the same in all series components.

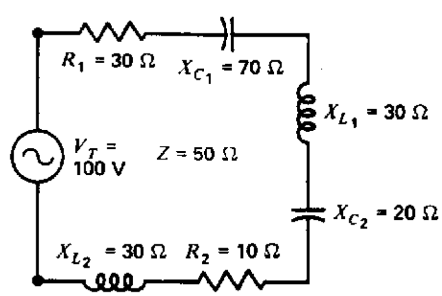


Figure 9
Series AC Circuit With More Components Than Figure 9, But The Same Values of *Z*, *I*, and θ

The total series X_C is 90 Ω, equal to the sum of 70 Ω for X_{C_1} and 20 Ω for X_{C_2}. Similarly, the total series X_L and 60 Ω. This value is equal to me sum of 30 Ω for X_{L_1} and 30 Ω for X_{L_2}.

The net reactance X equals 30 Ω, which is 90 Ω of X_C minus 60 Ω of X_L. Since X_C is larger than X_L, the net reactance is capacitive. The circuit in Figure 9 is equivalent to Figure 8, therefore, since a 40-Ω R is in series with a net X_C of 30 Ω.

Practice Problems — Section 5

a. In Figure 8, how much is the net reactance?

b. In Figure 9, how much is the net reactance?

Parallel Reactance and Resistance

With parallel circuits, the branch currents for resistance and reactance are added by phasors. Then the total line current is found by the formula

$$I_T = \sqrt{I_R^2 + I_X^2} \qquad (24\text{-}2)$$

Calculating I_T

As an example, Figure 10a shows a circuit with three branches. Since the voltage across all the parallel branches is the applied 100 V, the individual branch currents are

$$I_R = \frac{V_A}{R} = \frac{100\ \text{V}}{25\ \Omega} = 4\ \text{A}$$

$$I_L = \frac{V_A}{X_L} = \frac{100\ \text{V}}{25\ \Omega} = 4\ \text{A}$$

$$I_C = \frac{V_A}{X_C} = \frac{100\ \text{V}}{100\ \Omega} = 4\ \text{A}$$

The net reactive branch current I_X is 3 A, then, equal to the difference between the 4-A I_L and the 1-A I_C, as shown in Figure 10b.

The next step is to calculate I_T as the phasor sum of I_R and I_X. Then

$$
\begin{aligned}
I_T &= \sqrt{I_R^2 + I_X^2} \\
&= \sqrt{4^2 + 3^2} \\
&= \sqrt{16+9} = \sqrt{25} \\
I_T &= 5\text{ A}
\end{aligned}
$$

The phasor diagram for I_T is shown in Figure 10c.

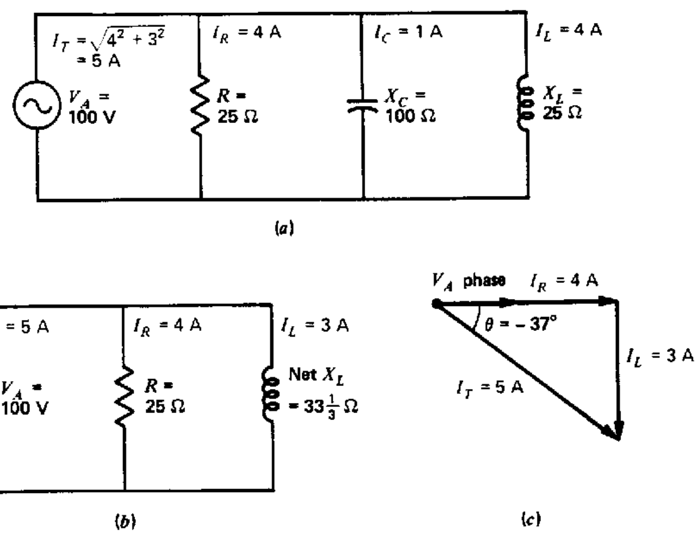


Figure 10
Total Line Current IT of Parallel Circuit
(a) Branches of *R*, *X_L*, and *X_C* in Parallel
(b) Equivalent Circuit with *I_R* and Net Reactive Branch Current
(c) Phasor Diagram

$Z_T = V_A / I_T$

This gives the total impedance of a parallel circuit. In this example, Z_T is 100 V/5 A, which equals 20Ω. This value is the equivalent impedance of all three branches in parallel across the source.

Phase Angle

The phase angle of the parallel circuit is found from the branch currents. Now θ is the angle whose tangent equals I_X/I_R.

For this example, I_X is the net inductive current of the 3-A I_L. Also, I_R is 4 A. These phasors are shown in Figure 10c. Then θ is a negative angle with the tangent of -3/4 or -0.75. This phase angle is -37°, approximately.

The negative angle for I_T indicates lagging inductive current. The value of -37° is the phase angle of I_T with respect to the voltage reference V_A.

When Z_T is calculated as V_A/I_T for a parallel circuit, the phase angle of Z_T is the same value as for I_T but with opposite sign. In this example, Z_T is 20 Ω with a phase angle of +37°, for an I_T of 5 A with an angle of -37°. We can consider that Z_T has the phase of the voltage source with respect to I_T.

More Parallel Branches

Figure 11 illustrates how any number of parallel resistances and reactances can be combined. The total resistive branch current I_R of 4 A is the sum of 2 A each for the R_1 branch and the R_2 branch. Note that the order of connection does not matter, since the parallel branch currents add in the main line. Effectively, two 50-Ω resistances in parallel are equivalent to one 25-Ω resistance.

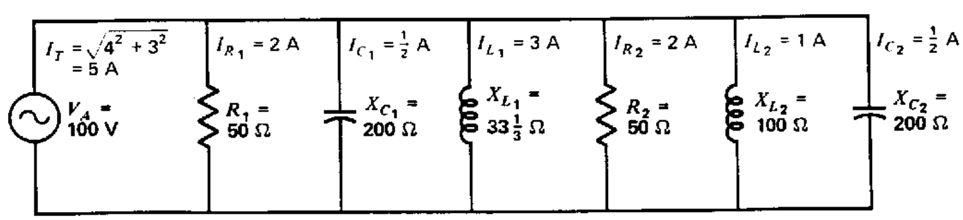


Figure 11

Parallel AC Circuit With More Components than Figure 10, But The Same Values of *Z, I,* and *θ*

Similarly, the total inductive branch current I_L is 4 A, equal to 3 A for I_{L_1} and 1 A for I_{L_2}. Also, the total capacitive branch current I_C is 1 A, equal to 1/2 A each for I_{C_1} and I_{C_2}.

The net reactive branch current I_X is 3 A, then, equal to a 4-A I_L minus a 1-A I_C. Since I_L is larger, the net current is inductive.

The circuit in Figure 11 is equivalent to the circuit in Figure 10, therefore. Both have a 4-A resistive current I_R and a 3-A net inductive current I_L. These values added by phasors make a total of 5 A for I_T in the main line.

Practice Problems — Section 6

a. In Figure 10, what is the net reactive branch current?

b. In Figure 11, what is the net reactive branch current.

Figure 12 shows how a series-parallel circuit can be reduced to a series circuit with just one reactance and one resistance. The method is straightforward as long as resistance and reactance are not combined in one parallel bank or series string.

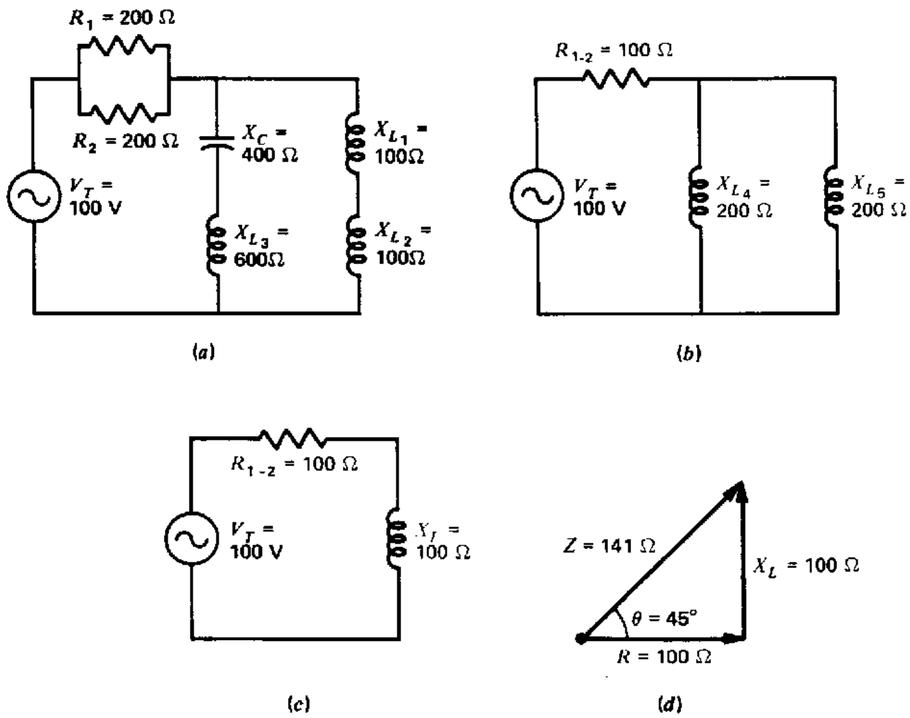


Figure 12
Reducing a Series-Parallel Circuit with *R*, X_L, and X_C to a Series Circuit With one *R* and One *X*. (a) Actual Circuit (b) Simplified Arrangement (c) Series Equivalent Circuit (d) Phasor Diagram

Working backward toward the generator from the outside branch in Figure 12a, we have an X_{L_1} and an X_{L_2} of 100 Ω each in series, which total 200 Ω. This string in Figure 12a is equivalent to X_{L_5} in Figure 12b.

In the other branch, the net reactance of X_{L_3} and X_C is equal to 600 Ω minus 400 Ω. This is equivalent to the 200 Ω of X_{L_4} in Figure 12b. The X_{L_4} and X_{L_5} of 200 Ω each in parallel are combined for an X_L of 100 Ω.

In Figure 12c, the 100-Ω X_L is in series with the 100-Ω $R_{1\text{-}2}$. This value is for R_1 and R_2 in parallel.

The phasor diagram for the equivalent circuit in Figure 12d shows the total impedance Z of 141 Ω for a 100-Ω R in series with a 100-Ω X_L.

With a 141-Ω impedance across the applied V_T of 100 V, the current in the generator is 0.7 A. The phase angle θ is 45° for this circuit.

Practice Problems — Section 7

Refer to Figure 12.

a. How much is $X_{L_1} + X_{L_2}$?

b. How much is X_{L_3} - X_C?

c. How much is X_{L_4} in parallel with X_{L_5}?

Real Power

In an ac circuit with reactance, the current I supplied by the generator either leads or lags the generator voltage V. Then the product VI is not the real power produced by the generator, since the voltage may have a high value while the current is near zero, or vice versa. The real power, however, can always be calculated as I^2R, where R is the total resistive component of the circuit, because current and voltage have the same phase in a resistance. To find the corresponding value of power as VI, this product must be multiplied by the cosine of the phase angle θ. Then

$$\text{Real power} = I^2R \qquad (24\text{-}3)$$

or

$$\text{Real power - } VI \cos \theta \qquad (24\text{-}4)$$

where V and I are in rms values, to calculate the real power, in watts. Multiplying VI by the cosine of the phase angle provides the resistive component for real power equal to I^2R.

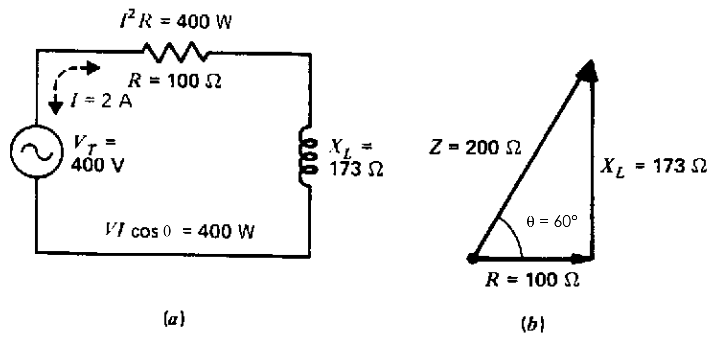


Figure 13
Real Power In Series Circuit.
(a) Schematic Diagram (b) Phasor Diagram

For example, the ac circuit in Figure 13 has 2 A through a 100-Ω R in series with the X_L of 173 Ω. Therefore

$$\text{Real power} = I^2R = 4 \times 100$$

$$\text{Real power} = 400 \text{ W}$$

Furthermore, in this circuit the phase angle is 60° with a cosine of 0.5. The applied voltage is 400 V. Therefore

$$\text{Real power} = VI \cos\theta = 400 \times 2 \times 0.5$$

$$\text{Real power} = 400 \text{ W}$$

In both examples, the real power is the same 400 W, because this is the amount of power supplied by the generator and dissipated in the resistance. Either formula can be used for calculating the real power, depending on which is more convenient.

Real power can be considered as resistive power, which is dissipated as heat. A reactance does not dissipate power but stores energy in the electric or magnetic field.

Power Factor

Because it indicates the resistive component, cos θ is the power factor of the circuit, converting the VI product to real power. For series circuits, use the formula

$$\text{Power factor} = \cos\theta = \frac{R}{Z}$$

or for parallel circuits

$$\text{Power factor} = \cos\theta = \frac{I_R}{I_T} \quad (24\text{-}6)$$

In Figure 13, as an example of a series circuit, we use R and Z for the calculations:

$$\text{Power factor} = \cos\theta = \frac{R}{Z} = \frac{100\,\Omega}{200\,\Omega} = 0.5$$

For the parallel circuit in Figure 10, we use the resistive current I_R and the I_T:

$$\text{Power factor} = \cos\theta = \frac{I_R}{I_T} = \frac{4\ A}{5\ A} = 0.8$$

The power factor is not an angular measure but a numerical ratio, with a value between 0 and 1, equal to the cosine of the phase angle.

With all resistance and zero reactance, R and Z are the same for a series circuit, or I_R and I_T are the same for a parallel circuit, and the ratio is 1. Therefore, unity power factor means a resistive circuit. At the opposite extreme, all reactance with zero resistance makes the power factor zero, meaning that the circuit is all reactive.

Apparent Power

When V and I are out of phase because of reactance, the power of V x I is called apparent power. The unit is voltamperes (VA) instead of watts, since the watt is reserved for real power.

For the example in Figure 13, with 400 V and the 2-A I, 60° out of phase, the apparent power is VI, or 400 x 2 = 800 VA. Note that apparent power is the VI product alone, without considering the power factor cos θ.

The power factor can be calculated as the ratio of real power to apparent power, as this ratio equals cos θ. As an example, in Figure 13, the real power is 400 W, and the apparent power is 800 VA. The ratio of $^{400}/_{800}$ then is 0.5 for the power factor, the same as cos 60°.

The VAR

This is an abbreviation for voltampere reactive. Specifically, VARs are voltamperes at the angle of 90°.

In general, for any phase angle θ between V and I, multiplying VI by $\sin \theta$ gives the vertical component at 90° for the value of the VARs. In Figure 13, the value of VI sin 60° is 800 x 0.866 = 692.8 VAR.

Note that the factor $\sin \theta$ for the VARs gives the vertical or reactive component of the apparent power VI. However, multiplying VI by $\cos \theta$ as the power factor gives the horizontal or resistive component for the real power.

Correcting the Power Factor

In commercial use, the power factor should be close to unity for efficient distribution. However, the inductive load of motors may result in a power factor of 0.7, as an example, for the phase angle of 45°. To correct for this lagging inductive component of the current in the main line, a capacitor can be connected across the line to draw leading current from the source. To bring the power factor up to 1.0, that is, unity PF, the value of capacitance is calculated to take the same amount of voltamperes as the VARs of the load.

Practice Problems — Section 8

a. What is the unit for real power?

b. What is the unit for apparent power?

AC Meters

The D'Arsonval moving-coil type of meter movement will not read if it is used in an ac circuit because the average value of an alternating current is zero. Since the two opposite polarities cancel, an alternating current cannot deflect the meter movement either up-scale or down-scale. An ac meter must produce deflection of the meter pointing up-scale regardless of polarity. This deflection is accomplished by one of the following three methods for ac meters.

1. *Thermal type*. In this method, the heating effect of the current, which is independent of polarity, is used to provide meter deflection. Two examples are the thermocouple type and hot-wire meter.

2. *Electromagnetic type*. In this method, the relative magnetic polarity is maintained constant although the current reverses. Examples are the iron-vane meter, dynamometer, and wattmeter.

3. *Rectifier type*. The rectifier changes the ac input to dc output for the meter, which is usually a D'Arsonval movement. This type is the most common for ac voltmeters generally used for the audio and radio frequencies.

All ac meters have scales calibrated in rms values, unless noted otherwise on the meter.

A thermocouple consists of two dissimilar metals joined together at one end but open at the opposite side. Heat at the short-circuited junction produces a small dc voltage across the open ends, which are connected to a dc meter movement. In the hot-wire meter, current heats a wire to make it expand, and this motion is converted into meter deflection. Both types are used as ac meters for radio frequencies.

The iron-vane meter and dynamometer have very low sensitivity, compared with a D'Arsonval movement. They are used in power circuits, for either direct current or 60-Hz alternating current.

Practice Problems — Section 9

Answer True or False.

a. The iron-vane meter can read alternating current.

b. The D'Arsonval meter movement is for direct current only.

The wattmeter uses fixed coils to indicate current in the circuit, while the movable coil indicates voltage (Figure 14). The deflection then is proportional to power. Either dc power or real ac power can be read directly by the wattmeter.

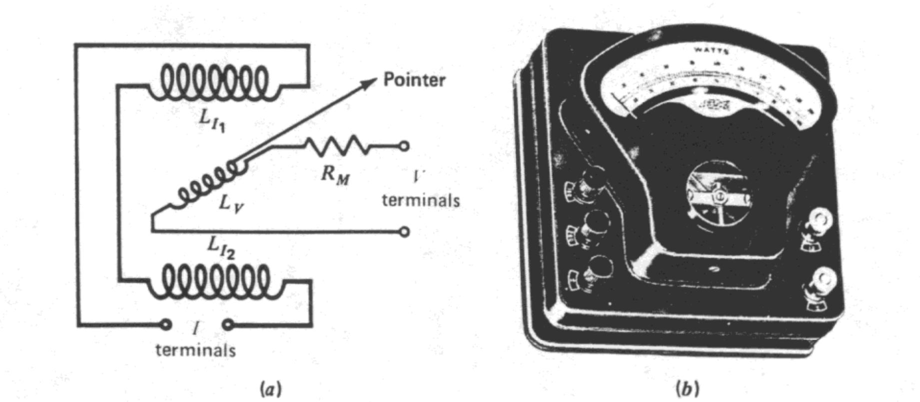


Figure 14
Wattmeter (a) Schematic of Voltage and Current Coils
(b) Meter For Range of 0 to 200 W. ***(W. M. Welch Mfg. Co.)***

In Figure 14a, the coils L_{I_1} and L_{I_2} in series are the stationary coils serving as an ammeter to measure current. The two I terminals are connected in one side of the line in series with the load. The movable coil L_V and its multiplier resistance R_M are used as a voltmeter, with the V terminals connected across the line in parallel with the load. Then the current in the fixed coils is proportional to I, while the current in the movable coil is proportional to V. As a result, the deflection is proportional to the VI product, which is power.

Furthermore, it is the VI product for each instant of time that produces deflection. For instance, if the V value is high when the I value is low, for a phase angle close to 90°, there will be little deflection. The meter deflection is proportional to the watts of real power, therefore, regardless of the power factor in ac circuits. The wattmeter is commonly used to measure power from the 60-Hz power line. For radio frequencies, however, power is generally measured in terms of heat transfer.

a. Does a wattmeter measure real or apparent power?

b. In Figure 14, does the movable coil of the wattmeter measure V or I?

Summary

The differences in R, X_L, X_C, and Z are listed in the Table 1, but the following general features should also be noted. Ohms of opposition limit the amount of current in dc circuits or ac circuits. Resistance R is the same for either case. However, ac circuits can have ohms of reactance because of the variations in alternative current or voltage. Reactance X_L is the reactance of an inductance with sine-wave changes in current. Reactance X_C is the reactance of a capacitor with sine-wave changes in voltage.

	Resistance R, Ω	Inductive reactance X_L, Ω	Capacitive reactance X_C, Ω	Impedance Z, Ω
Definition	In-phase opposition to alternating or direct current	90° leading opposition to alternating current	90° lagging opposition to alternating current	Phasor combination of resistance and reactance $Z = \sqrt{R^2 + X^2}$
Effect of frequency	Same for all frequencies	Increases with higher frequencies	Decreases with higher frequencies	X_L component increases, but X_C decreases
Phase angle θ	0°	I_L lags V_L by 90°	V_C lags I_C by 90°	$\tan\theta = \pm\frac{X}{R}$ in series, or $\pm\frac{I_X}{I_R}$ in parallel

Table 1
Types of Ohms in AC Circuits

Both X_L and X_C are measured in ohms, like R, but reactance has a 90° phase angle, while the phase angle for resistance is 0°. A circuit with steady direct current cannot have any reactance.

Ohms of X_L or X_C are opposite, as X_L has a phase angle of +90°, while X_C has the angle of -90°. Any individual X_L or X_C always has a phase angle that is exactly 90°.

Ohms of impedance Z result from the phasor combination of resistance and reactance. In fact, Z can be considered the general form of any ohms of opposition in ac circuits.

Z can have any phase angle, depending on the relative amounts of R and X. When Z consists mostly of R with little reactance, the phase angle of Z is close to 0°. With R and X equal, the phase angle of Z is 45°. Whether the angle is positive or negative depends on whether the net reactance is inductive or capacitive. When Z consists mainly of X with little R, the phase angle of Z is close to 90°.

The phase angle is θ_Z for Z or V_T with respect to the common I in a series circuit. With parallel branch currents, θ_I is for I_T in the main line with respect to the common voltage.

Practice Problems Section 11

a. Which of the following does not change with frequency: Z, X_L, X_C, or R?

b. Which has lagging current: R, X_L, or X_V?

c. Which has leading current: R, X_L, or X_V?

Summary of Types of Phasors in AC Circuits

The phasors for ohms, volts, and amperes are shown in Figure 15. Note the similarities and differences:

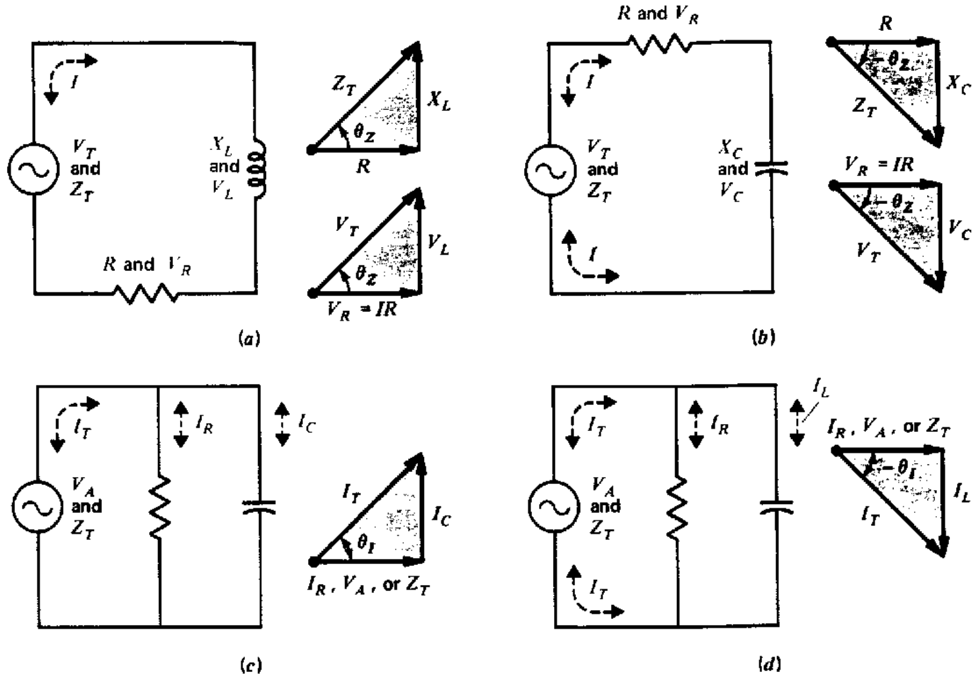


Figure 15
Summary of Phasor Relations in AC Circuits
(a) Reactance X_L and R in Series (b) Reactance X_C and R in Series
(c) Parallel Branches with I_C and I_R (d) Parallel Branches With I_L and I_R

Series Components In series circuits, ohms and voltage drops have similar phasors. The reason is the common I for all the series components. Therefore:

V_R or IR has the same phase as R.

V_L or IX_L has the same phase as X_L.

V_C or IX_C has the same phase as X_C.

Resistance

The R, V_L, and I_R always have the same angle because there is no phase shift in a resistance. This applies to R in either a series or a parallel circuit.

Reactance

Reactances X_L and X_C are 90° phasors in opposite directions. The X_L or V_L has the angle of +90° with an upward phasor, while the X_C or V_C has the angle of -90° with a downward phasor.

Reactive Branch Currents

The phasor of a parallel branch current is opposite from its reactance. Therefore, I_C is upward at +90°, opposite from X_C downward at -90°. Also, I_L is downward at -90°. opposite from X_L upward at +90°.

In short, I_C and I_L are opposite from each other, and both are opposite from their corresponding reactances.

Phase Angel θ_Z

The phasor resultant for ohms of reactance and resistance is the impedance Z. The phase angle θ for Z can be any angle between 0 and 90°. In a series circuit θ_Z for Z is the same as θ for V_T with respect to the common current I.

Phase Angle θ_I

The phasor resultant of branch currents is the total line current I_T. The phase angle of I_T can be any angle between 0 and 90°. In a parallel circuit, θ_I is the angle of I_T with respect to the applied voltage V_A.

The θ_I is the same value but of opposite sign from θ_Z for Z, which is the impedance of the combined parallel branches.

The reason for the change of sign is that θ_I is for I_T with respect to the common V, but θ_Z is for V_T with respect to the common current I.

Such phasor combinations are necessary in sine-wave ac circuits in order to take into account the effect of reactance. The phasors can be analyzed either graphically, as in Figure 15, or by the shorter technique of complex numbers, with a *j* operator that corresponds to a 90° phasor.

Practice Problems — Section 12

a. Of the following three phasors, which two are 180° opposite: V_L, V_C, or V_R?

b. Of the following three phasors, which two are out of phase by 90°: I_R, I_T, or I_L?

1. In ac circuits with resistance alone, the circuit is analyzed the same way as for dc circuits, generally with rms ac values. Without any reactance, the phase angle is zero.
2. When capacitive reactances alone are combined, the X_C values are added in series and combined by the reciprocal formula in parallel, just like ohms of resistance. Similarly, ohms of X_L alone can be added in series or combined by the reciprocal formula in parallel, just like ohms of resistance.
3. Since X_C and X_L are opposite reactances, they cancel each other. In series, the ohms of X_C and X_L cancel. In parallel, the capacitive and inductive branch currents I_C and I_L cancel.
4. In ac circuits with R, X_L, and X_C, they can be reduced to one equivalent resistance and one net reactance.
5. In series, the total R and net X at 90° are combined as $Z = \sqrt{R^2 + X^2}$. The phase angle of the series R and X is the angle with tangent ±X/R. First we calculate Z_T and then divide into V_T to find I.
6. For parallel branches, the total I_R and net reactive I_X at 90° are combined as $I_T = \sqrt{I_R^2 + I_X^2}$. The phase angle of the parallel R and X is the angle with tangent $\pm I_X/I_R$. First we calculate I_T and then divide into V_A to find Z_T.
7. The quantities R, X_L, X_C, and Z in ac circuits all are ohms of opposition. The differences with respect to frequency and phase angle are summarized in Table 1.
8. The phasor relations for resistance and reactance are summarized in Figure 15.
9. In ac circuits with reactance, the real power in watts equals I^2R, or VI cos θ, where θ is the phase angle. The real power is the power dissipated as heat in resistance. Cos θ is the power factor of the circuit.
10. The wattmeter measures real ac power or dc power.

Choose (a), (b), (c), or (d).

1. In an ac circuit with resistance but no reactance, (a) two 1000-Ω resistances in series total 1414 Ω; (b) two 1000-Ω resistances in series total 2000 Ω; (c) two 1000-Ω resistances in parallel total 707 Ω; (d) a 1000-Ω R in series with a 400-Ω R totals 600 Ω.
2. An ac circuit has an 100-Ω X_{C_1}, a 50-Ω X_{C_2}, a 40-Ω X_{L_1}, and a 30-Ω X_{L_2}, all in series. The net reactance is equal to (a) an 80-Ω X_L; (b) a 200-Ω X_L; (c) an 80-Ω X_C; (d) a 200-Ω X_C.
3. An ac circuit has a 40-Ω R, a 90-Ω X_L, and a 60-Ω X_C, all in series. The impedance Z equals (a) 50 Ω; (b) 70.7 Ω; (c) 110 Ω; (d) 190 Ω.
4. An ac circuit has a 100-Ω R, a 100-Ω X_L, and a 100-Ω X_C, all in series. The impedance Z of the series combination is equal to (a) 33-1/3 Ω; (b) 70.7 Ω; (c) 100 Ω; (d) 300 Ω.
5. An ac circuit has a 100-Ω R, a 300-Ω X_L, and a 200-Ω X_C, all in series. The phase angle θ of the circuit equals (a) 0°; (b) 37°; (c) 45°; (d) 90°.
6. The power factor of an ac circuit equals (a) the cosine of the phase angle: (b) the tangent of the phase angle; (c) zero for a resistive circuit; (d) unity for a reactive circuit.
7. Which phasors in the following combinations are not in opposite directions? (a) X_L and X_C; (b) X_L and I_C; (c) I_L and I_C; (d) X_C and I_C.
8. In Figure 8a, the voltage drop across X_L equals (a) 60 V; (b) 66-2/3 V; (c) 120 V; (d) 200 V.
9. In Figure 10a, the combined impedance of the parallel circuit equals (a) 5 Ω; (b) 12.5 Ω; (c) 20 Ω; (d) 100 Ω.
10. The wattmeter (a) has voltage and current coils to measure real power; (b) has three connections, two of which are used at a time; (c) measures apparent power because the current is the same in the voltage and current coils; (d) can measure dc power but not 60-Hz ac power.

1. Why can series or parallel resistances be combined in ac circuits the same way as in dc circuits?
2. (a) Why do X_L and X_C reactances in series cancel each other? (b) With X_L and X_C reactances in parallel, why do their branch currents cancel?
3. Give one difference in electrical characteristics comparing R and X_C, R and Z, X_C and C, X_L and L.
4. Name three types of ac meters.
5. Make a diagram showing a resistance R_1 in series with the load resistance R_L, with a wattmeter connected to measure the power in R_L.
6. Make a phasor diagram for the circuit in Figure 8a showing the phase of the voltage drops IR, IX_C, and IX_L with respect to the reference phase of the common current I.
7. Explain briefly why the two opposite phasors at +90° for X_L and -90° for I_L both follow the principle that any self-induced voltage leads the current through the coil by 90°.
8. Why is it that a reactance phasor is always at exactly 90° but an impedance phasor can be less than 90°?
9. Why must the impedance of a series circuit be more than either its X or R?
10. Why must I_T in a parallel circuit be more than either I_R or I_X?

1. Refer to Figure 1a. (a) Calculate the total real power supplied by the source. (b) Why is the phase angle zero? (c) What is the power factor of the circuit?

2. In a series ac circuit, 2 A flows through a 20-Ω R, a 40-Ω X_L, and a 60-Ω X_C. (a) Make a schematic diagram of the series circuit. (b) Calculate the voltage drop across each series component. (c) How much is the applied voltage? (d) Calculate the power factor of the circuit. (e) What is the phase angle θ?

3. A parallel circuit has the following five branches: three resistances of 30 Ω each; an X_L of 600 Ω; and X_C of 400 Ω. (a) Make a schematic diagram of the circuit. (b) If 100 V is applied, how much is the total line current? (c) What is the total impedance of the circuit? (d) What is the phase angle θ?

4. Referring to Figure 8, assume that the frequency is doubled from 500 to 1000 Hz. Find X_L, X_C, Z, I, and θ for this higher frequency. Calculate L and C.

5. A series circuit has a 300-Ω R, a 500-Ω X_{C_1}, a 300-Ω X_{C_2}, an 800-Ω X_{L_1}, and 400-Ω X_{L_2}, all in series with an applied voltage V of 400 V. (a) Draw the schematic diagram with all components. (b) Draw the equivalent circuit reduced to one resistance and one reactance. (c) Calculate Z_T, I, and θ.

6. Repeat Prob. 5 for a circuit with the same components in parallel across the voltage source.

7. A series circuit has a 600-Ω R, a 10-μH inductance L, and a 4-μF capacitance C, all in series with the 60-Hz 120-V power line as applied voltage. (a) Find the reactance of L and of C. (b) Calculate Z_T, I, and θ_Z.

8. Repeat Prob. 7 for the same circuit, but the 120-V source has f = 10 Mhz.

9. (a) Referring to the series circuit Figure 6, what is the phase angle between the IX_L voltage of 360 V and the IX_C voltage of 240 V? (b) Draw the two sine waves for these voltages, showing their relative amplitudes and phase corresponding to the phasor diagram in Figure 6b. Also show the resultant sine wave of voltage across the net X_L.

10. How much resistance dissipates 600 W of ac power, with 4.3-A rms current?

11. How much resistance must be inserted in series with a 0.95-H inductance to limit the current to 0.25 A from the 120-V 60-Hz power line?

12. How much resistance must be inserted in series with a 10-μF capacitance to provide a phase angle of -45°? The source is the 120-V 60-Hz power line.

13. With the same R as in Prob. 12, what value of C is necessary for the angle of -45° at the frequency of 2 Mhz?

14. A parallel ac circuit has the following branch currents: I_{R_1} = 4.2 mA; I_{R_2} = 2.4 mA; I_{L_1} = 7 mA; I_{L_2} = 1 mA; I_C = 6 mA. Calculate I_T.

15. With 420 mV applied, an ac circuit has the following parallel branches: R_1 = 100 Ω; R_2 = 175 Ω; X_{L_1} = 60 Ω; X_{L_2} = 420 Ω; X_C = 70 Ω. Calculate I_T, θ_I, and Z_T.

16. The same components as in Prob. 15 are in series. Calculate Z_T, I, and θ_Z.

17. What R is needed in series with a 0.01-μF capacitor for a phase angle of -64°, with f of 800 Hz?

Section 1	a. 0°
	b. 0°
Section 2.	a. 90°
	b. -90°
Section 3	a. -90°
	b. 90°
Section 4	a. 20 Ω
	b. 1 A
Section 5	a. X_C = 30 Ω
	b. X_C = 30 Ω
Section 6	a. I_L = 3 A
	b. I_L = 3 A
Section 7	a. 200 Ω
	b. 200 Ω
	c. 100 Ω
Section 8	a. Watt
	b. Voltampere
Section 9	a. T
	b. T
Section 10	a. Real power
	b. V
Section 11	a. R
	b. X_L
	c. X_C
Section 12	a. V_L and V_C
	b. I_R and I_L

1. (a) 100 W
   (b) No reactance
   (c) 1
2.
3. (b) $I = 10$ A, approx.
   (c) $Z = 10\ \Omega$
   (d) $\theta = 0°$
4.
5. (c) $Z_T = 500\ \Omega$
   $I = 0.8$ A
   $\theta_Z = 53°$
6.
7. (a) $X_L = 0$, approx.
   $X_C = 665\ \Omega$
   (b) $Z_T = 890\ \Omega$
   $I = 135$ mA
   $\theta_Z = -47.9°$
8.
9. (a) 180°
10.
11. $R = 102\ \Omega$
12.
13. $C = 300$ pF
14.
15. $I_T = 6.9$ mA, $\theta_I = -16.9°$
    $Z_T = 60.9\ \Omega$, $\theta_Z = 16.9°$
16.
17. $R = 9704\ \Omega$

Complex Numbers for AC Circuits

Complex numbers form a numerical system that includes the phase angle of a quantity, with its magnitude. Therefore, complex numbers are useful in ac circuits when the reactance of X_L or X_C makes it necessary to consider the phase angle.

Any type of ac circuit can be analyzed with complex numbers, but they are especially convenient for solving series-parallel circuits that have both resistance and reactance in one or more branches. Actually, the use of complex numbers is probably the best way to analyze ac circuits with series-parallel impedances.

Important terms in this chapter are:

admittance
imaginary numbers
j operator
polar form
real numbers
rectangular form
susceptance

More details are explained in the following sections:

1. Positive and Negative Numbers
2. The *j* Operator
3. Definition of a Complex Number
4. How Complex Numbers are Applied to AC Circuits
5. Impedance in Complex form
6. Operations with Complex Numbers
7. Magnitude and Angle of a Complex Number
8. Polar Form of Complex Numbers
9. Converting Polar to Rectangular Form
10. Complex Numbers in Series AC Circuits
11. Complex Numbers in Parallel AC Circuits
12. Combining Two Complex Branch Impedances
13. Combining Complex Branch Currents
14. Parallel Circuit with Three Complex Branches

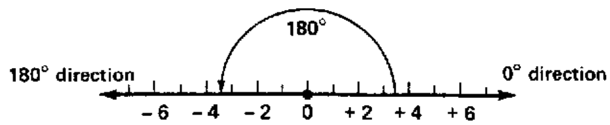


Figure 1
Positive and Negative Numbers

Our common use of numbers as either positive or negative represents only two special cases. In their more general form, numbers have both quantity and phase angle. In Figure 1, positive and negative numbers are shown as corresponding to the phase angles of 0 and 180°, respectively.

For example, the numbers 2, 4, and 6 represent units along the horizontal or x axis, extending toward the right along the line of zero phase angle. Therefore, positive numbers really represent units having the phase angle of 0°. Or this phase angle corresponds to the factor of +1. To indicate 6 units with zero phase angle, then, 6 is multiplied by +1 as a factor for the positive number 6. The + sign is often omitted, as it is assumed unless indicated otherwise.

In the opposite direction, negative numbers correspond to 180°. Or, this phase angle corresponds to the factor of -1. Actually, -6 represents the same quantity as 6 but rotated through the phase angle of 180°. The angle of rotation is the operator for the number. The operator for -1 is 180°; the operator for +1 is 0°.

Practice Problems — Section 1

a. What is the angle for the number +5?

a. What is the angle for the number -5?

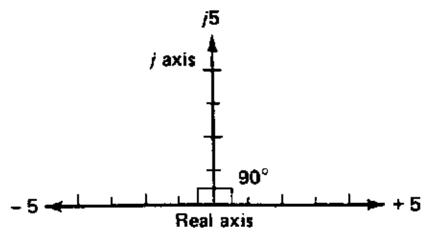


Figure 2
The *j* Axis at 90° From Real Axis

The operator for a number can be any angle between 0 and 360°. Since the angle of 90° is important in ac circuits, the factor *j* is used to indicate 90°. See Figure 2. Here, the number 5 means 5 units at 0°, the number -5 is at 180°, while j5 indicates the 90° angle.

The *j* is usually written before the number. The reason is that the *j* sign is a 90° operator, just as the + sign is a 0° operator and the - sign is a 180° operator. Any quantity at right angles to the zero axis, therefore, 90° counterclockwise, is on the +*j* axis.

In mathematics, numbers on the horizontal axis are real numbers, including positive and negative values. Numbers on the *j* axis are called *imaginary numbers*, only because they are not on the real axis. Also, in mathematics the abbreviation I is used in place of *j*. In electricity, however, *j* is used to avoid confusion with I as the symbol for current. Furthermore, there is nothing imaginary about electrical quantities on the *j* axis. An electric shock from j500 V is just as dangerous as 500 V positive or negative.

More featured of the *j* operator are shown in Figure 3. The angle of 180° corresponds to the *j* operation of 90° repeated twice. This angular rotation is indicated by the factor j^2. Note that the *j* operation multiplies itself, instead of adding.

Since j^2 means 180°, which corresponds to the factor of -1, we can say that j^2 is the same as -1. In short, the operator j^2 for a number means multiply by -1. For instance, $j^2 8$ is -8.

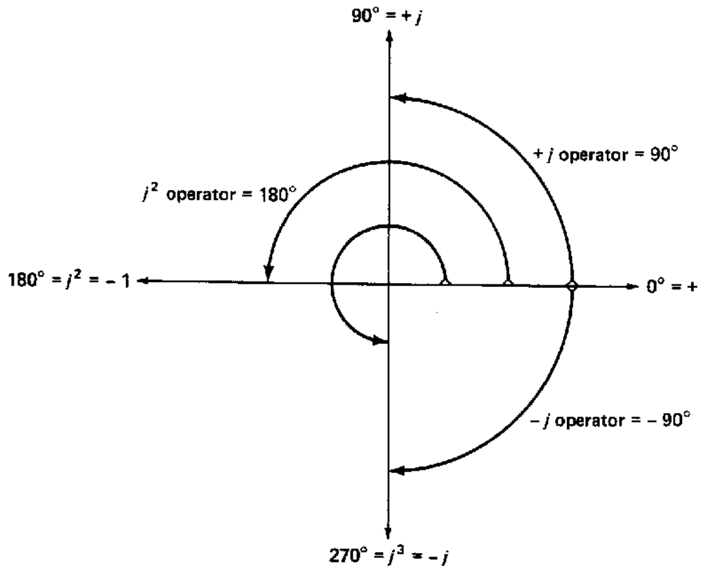


Figure 3

Furthermore, the angle of 270° is the same as -90°, which corresponds to the operator *-j*. These characteristics of the *j* operator are summarized as follows:

$$0° = 1$$

$$90° = j$$

$$180° = j^2 = -1$$

$$270° = j^3 = j^2 \times j = -1 \times j = -j$$

$$360° = \text{same as } 0°$$

As examples, the number 4 or -4 represents 4 units on the real horizontal axis; j4 means 4 units with a leading phase angle of 90°; -j4 means 4 units with a lagging phase angle of -90°.

Practice Problems — Section 2

a. What is the angle for the operator *j*?

b. What is the angle for the operator *-j*?

The combination of a real and imaginary term is a complex number. Usually, the real number is written first. As an example, 3 + j4 is a complex number including 3 units on the real axis added to 4 units 90° out of phase on the *j* axis. The name *complex number* just means that its terms must be added as phasors.

Phasors for complex numbers are shown in Figure 4. The +*j* phasor is up for 90°; the -*j* phasor is down for -90°. The phasors are shown with the end of one joined to the start of the next, to be ready for addition. Graphically, the sum is the hypotenuse of the right triangle formed by the two phasors. Since a number like 3 + j4 specifies the phasors in rectangular coordinates, this system is the *rectangular form* of complex numbers.

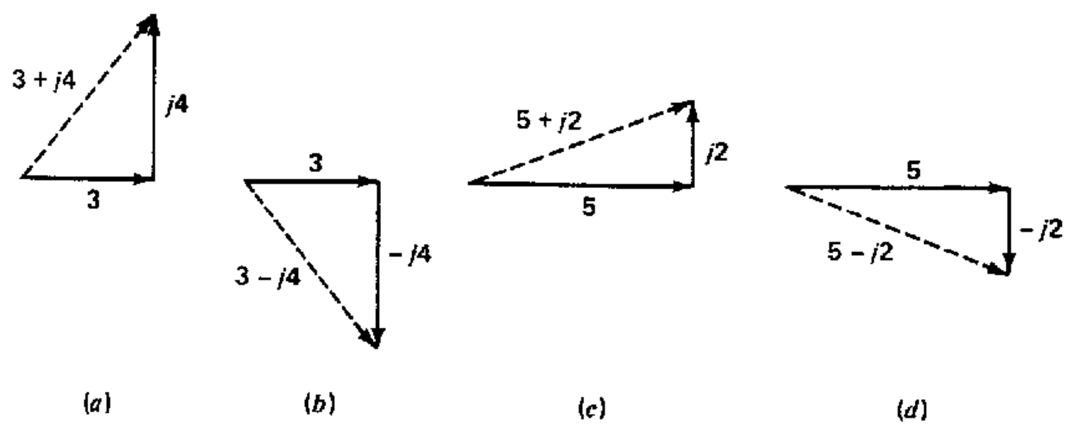


Figure 4
Phasors Corresponding to Real Terms and *j* Terms, In Rectangular Coordinates

Be careful to distinguish a number like j2, where 2 is a coefficient, from j^2, where 2 is the exponent. The number j2 means 2 units up on the *j* axis of 90°. However, j^2 is the operator of -1, which is on the real axis in the negative direction.

Another comparison to note is between j3 and j^3. The number j3 is 3 units up on the *j* axis, while j^3 is the same as the -*j* operator, which is down on the -90° axis.

Also note that either the real term or *j* term can be the larger of the two. When the *j* term is larger, the angle is more than 45°; when the *j* term is smaller, the angle is less than 45°. If the *j* term and the real term are equal, the angle is 45°.

Practice Problems — Section 3

Answer True or False.

a. For 7 + j6, the 6 is at 90° leading the 7.
b. For 7 - j6, the 6 is at 90° lagging the 7.

How Complex Numbers Are Applied to AC Circuits

The applications are just a question of using a real term for 0°, +*j* for 90°, and -*j* for -90°, to denote the phase angles. Specifically, Figure 5 below illustrates the following rules:

An angle of 0° or a real number without any *j* operator is used for resistance R. For instance, 3 Ω of R is stated just as 3 Ω.

An angle of 90° or +*j* is used for inductive reactance X_L. For instance, a 4-Ω X_L is j4 Ω. This rule always applies to X_L, whether it is in series or parallel with R. The reason is the fact that X_L represents voltage across an inductance, which always leads the current through the inductance by 90°. The +*j* is also used for V_L.

An angle of -90° or -*j* is used for capacitive reactance X_C. For instance, a 4-Ω X_C is -j4 W. This rule always applies to X_C, whether it is in series or parallel with R. The reason is the fact that X_C represents voltage across a capacitor, which always lags the charge and discharge current of the capacitor by -90°. The -*j* is also used for V_C.

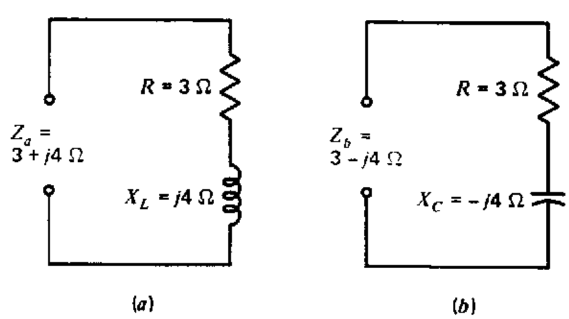


Figure 5
Rectangular Form of Complex Numbers For Impedances
(a) Reactance X_L is +*J* (b) Reactance XC is -*j*

With reactive branch currents, the sign for *j* is reversed, compared with reactive ohms, because of the opposite phase angle. As shown in Figure 6a and b, -*j* is used for inductive branch current I_L and +*j* for capacitive branch current I_C.

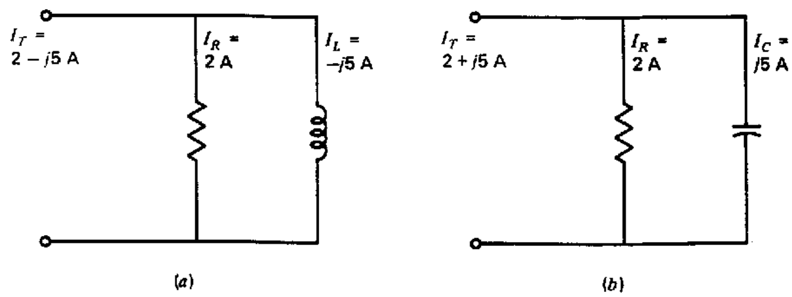


Figure 6
Rectangular Form of Complex Numbers For Branch Currents
(a) Current I_L is -*j*   (b) Current L_C is + *j*

Practice Problems — Section 4

a. Write 3 kΩ of K_L with the *j* operator.

b. Write 5 mA of I_L with the *j* operator.

Impedance in Complex Form

The rectangular form of complex numbers is a convenient way to state the impedance of series resistance and reactance. In Figure 5a, the impedance is 3 + j4, as Z_a is the phasor sum of a 3-Ω R in series with j4 Ω for X_L. Similarly, Z_b is 3 - j4 for a 3-Ω R in series with -j4 Ω for X_C. The minus sign results from adding the negative term for -*j*. More examples are:

For a 4-kΩ R and a 2-kΩ X_L in series,

$$Z_T = 4000 + j2000$$

For a 3-kΩ R and a 9-kΩ X_C in series,

$$Z_T = 3000 - j9000$$

For a zero R and a 7-Ω XL in series,

$$Z_T = 0 + j7$$

For a 12-Ω R and a zero reactance in series,

$$Z_T = 12 + j0$$

Note the general form of stating Z = R ± jX. If one term is zero, substitute 0 for this term, in order to keep Z in its general form. This procedure is not required, but there is usually less confusion when the same form is used for all types of Z.

The advantage of this method is that multiple impedances written as complex numbers can then be calculated as follows:

$$Z_T = Z_1 + Z_2 + Z_3 + \ldots + \text{etc.}$$

for series impedances

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots + \text{etc.}$$

for parallel impedances

or

$$Z_T = \frac{Z_1 \text{ x } Z_2}{Z_1 + Z_2}$$ for two parallel impedances

Examples are shown in Figure 7. The circuit in Figure 7a is just a series combination of resistances and reactances. Combining the real terms and *j* terms separately, $Z_T = 12 + j4$. The calculations are 3 + 9 = 12 Ω for R and j6 added to -j2 equals j4 for the next X_L.

The parallel circuit in Figure 7b shows that X_L is +*j* and X_C is -*j* even though they are in parallel branches, as they are reactances, not currents.

So far, these types of circuits can be analyzed with or without complex numbers. For the series-parallel circuit in Figure 7c, however, the notation of complex numbers is necessary to state the complex impedance Z_T, consisting of branches with reactance and resistance in one or more of the branches. Impedance Z_T is just stated here in its form as a complex impedance. In order to calculate Z_T, some of the rules described in the next section must be used for combining complex numbers.

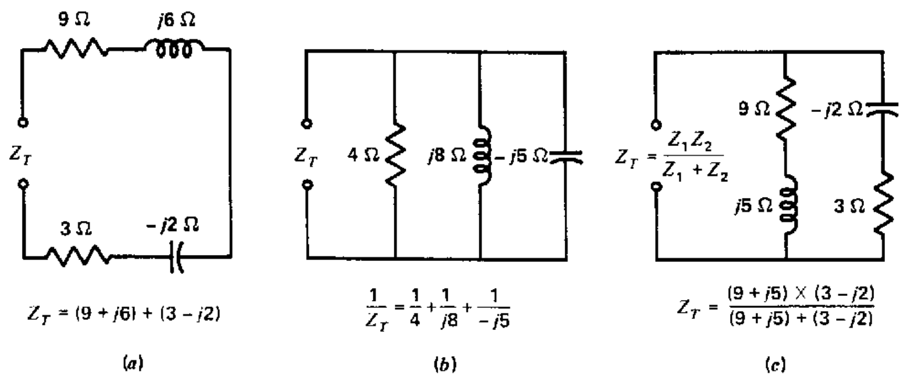


Figure 7
Reactance X_L is a +*j* Term and X_C is a -*j* Term, Whether in Series or in Parallel (a) Series Circuit (b) Parallel Branches (c) Complex Branch Impedances Z_1 and Z_2 in Parallel

Practice Problems — Section 5

Write the following impedances in complex form.

a. X_L of 7 Ω in series with R of 4 Ω.

b. X_C of 7 Ω in series with zero R.

Operations with Complex Numbers

Real numbers and *j* terms cannot be combined directly because they are 90° out of phase. The following rules apply:

For Addition or Subtraction

Add or subtract the real and *j* terms separately:

$$(9 + j5) + (3 + j2) = 9 + 3 + j5 + j2 = 12 + j7$$

$$(9 + j5) + (3 - j2) = 9 + 3 + j5 - j2 = 12 + j3$$

$$(9 + j5) + (3 - j8) = 9 + 3 + j5 - 8 = 12 - j3$$

The answers should be in the form of R ± jX, where R is the algebraic sum of all the real or resistive terms and X is the algebraic sum of all the imaginary or reactive terms.

To Multiply or Divide a j Term by a Real Number

Just multiply or divide the numbers. The answer is still a *j* term. Note the algebraic signs in the following examples. If both factors have the same sign, either + or -, the answer is +; if one factor is negative, the answer is negative.

4 x j3 = j12	j12 ÷ 4 = j3
j5 x 6 = j30	j30 ÷ 6 = j5
j5 x (-6) = -j30	-j30 ÷ (-6) = j5
-j5 x 6 = -j30	-j30 ÷ 6 = -j5
-j5 x (-6) = j30	j30 ÷ (-6) = -j5

To Multiply or Divide a Real Number by a Real Number

Just multiply or divide the real numbers, as in arithmetic. There is no *j* operation. The answer is still a real number.

To Multiply a j Term by a j Term

Multiply the numbers and the *j* coefficients to produce a j^2 term. The answer is a real term because j^2 is -1, which is on the real axis. Multiplying two *j* terms shifts the number 90° from the *j* axis to the real axis of 180°. As examples:

$$j4 \times j3 = j^2 12 = (-1)(12)$$
$$= -12$$
$$j4 \times (-j3) = -j^2 12 = -(-1)(12)$$
$$= 12$$

To Divide a j Term by a j Term

Divide the *j* coefficients to produce a real number: the *j* factors cancel. For instance:

j12 ÷ j4 = 3	-j12 ÷ j4 = -3

$j30 \div j5 = 6$

$j15 \div j3 = 5$

$j30 \div (-j6) = -5$

$-j15 \div (-j3) = 5$

To Multiply Complex Numbers

Follow the rules of algebra for multiplying two factors, each having two terms:

$$
\begin{aligned}
(9 + j5) \times (3 - j2) &= 27 + j15 - j18 - j^2 10 \\
&= 27 - j3 - (-1)10 \\
&= 27 - j3 + 10 \\
&= 37 - j3
\end{aligned}
$$

Note that $-j^2 10$ equals +10 because the operator j^2 is -1 and -(-1)10 becomes +10.

To Divide Complex Numbers

This process becomes more involved because division of a real number by an imaginary number is not possible. Therefore, the denominator must first be converted to a real number without any *j* term.

Converting the denominator to a real number without any *j* term is called *rationalization* of the fraction. To do this, multiply both numerator and denominator by the *conjugate* of the denominator. Conjugate complex numbers have equal terms but opposite signs for the *j* term. For instance, (1 + j2) has the conjugate (1 - j2).

Rationalization is permissible because the value of fraction is not changed when both numerator and denominator are multiplied by the same factor. This procedure is the same as multiplying by 1. In the following example of division with rationalization the denominator (1 + j2) has the conjugate (1 - j2):

$$
\begin{aligned}
\frac{4 - j1}{1 + j2} &= \frac{4 - j1}{1 + j2}\, x\, \frac{(1 - j2)}{(1 - j2)} \\
&= \frac{4 - j8 - j1 + j^2 2}{1 - j^2 4} \\
&= \frac{4 - j9 - 2}{1 + 4} \\
&= \frac{2 - j9}{5} \\
&= 0.4 - j1.8
\end{aligned}
$$

As a result of the rationalization, 4 - j1 has been divided by 1 + j2 to find the quotient that is equal to 0.4 - j1.8.

Note that the product of a complex number and its conjugate always equals the sum of the squares of the numbers in each term. As another example, the product of (2 + j3) and its conjugate (2 - j3) must be 4 + 9, which equals 13. Simple numerical examples of division and multiplication are given here because when the required calculations become too long, it is easier to divide and multiply complex numbers in polar form, as explained in Section 8.

Practice Problems — Section 6

a. (2 + j3) + (3 + j4) = ?

b. (2 + j3) x 2 = ?

Magnitude and Angle of a Complex Number

In electrical terms a complex impedance (4 + j3) means 4 Ω of resistance and 3 Ω of inductive reactance with a leading phase angle of 90°. See Figure 8a. The magnitude of Z is the resultant, equal to $\sqrt{16+9} = \sqrt{25} = 5$ Ω. Finding the square root of the sum of the squares is vector or phasor addition of two terms in quadrature, 90° out of phase.

The phase angle of the resultant is the angle whose tangent is 3/4 or 0.75. The angle equals 37°. Therefore, 4 + j3 = 5 $\angle 37°$.

When calculating the tangent ratio, note that the *j* term is the numerator and the real term is the denominator because the tangent of the phase angle is the ratio of the opposite side to the adjacent side. With a negative *j* term, the tangent is negative, which means a negative phase angle.

Note the following definitions: (4 + j3) is the complex number in rectangular coordinates. The real term is 4. The imaginary term is j3. The resultant 5 is the magnitude, absolute value, or modulus of the complex number. Its phase angle or argument is 37°. The resultant value by itself can be written as |5|, with vertical lines to indicate it is the magnitude without the phase angle. The magnitude is the value a meter would read.

For instance, with a current of $5\angle 37°$ A in a circuit, an ammeter reads 5 A. As additional examples:

$$2 + j4 = \sqrt{4 + 16}\ (\text{arctan } 2) = 4.47\angle 63°$$

$$4 + j2 = \sqrt{16 + 4}\ (\text{arctan } 0.5) = 4.47\angle 26.5°$$

$$8 + j6 = \sqrt{64 + 36}\ (\text{arctan } 0.75) = 10\angle 37°$$

$$8 - j6 = \sqrt{64 + 36}\ (\text{arctan } -0.75) = 10\angle -37°$$

$$4 + j4 = \sqrt{16 + 16}\ (\text{arctan } 1) = 5.66\angle 45°$$

$$4 - j4 = \sqrt{16 + 16}\ (\text{arctan } -1) = 5.66\angle -45°$$

Note that arctan 2, for example, means the angle with a tangent equal to 2. This can also be indicated as $\tan^{-1} 2$. In either case, the angle is specified as having 2 for its tangent, and the angle is 63.4°.

Practice Problems — Section 7

For the complex impedance 10 + j10 Ω.

a. Calculate the magnitude.

b. Calculate the phase angle.

Polar Form of Complex Numbers

Calculating the magnitude and phase angle of a complex number is actually converting to an angular form in polar coordinates. As shown in Figure 8, the rectangular form 4 + j3 is equal to $5\angle 37°$ in polar form. In polar coordinates, the distance out from the center is the magnitude of the vector Z. Its phase angle θ is counterclockwise from the 0° axis.

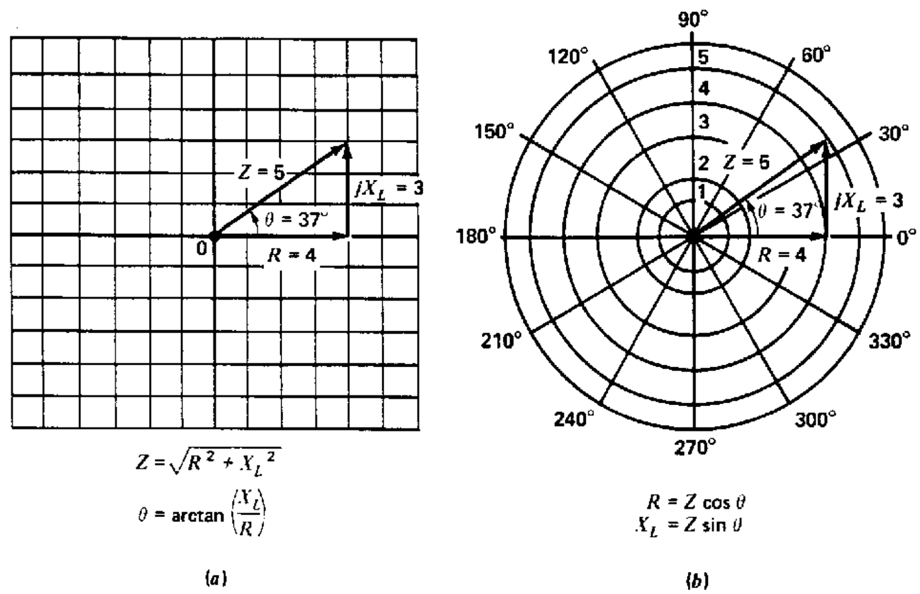


Figure 8
Magnitude and Angle of a Complex Number
(a) Rectangular Form (b) Polar Form

To convert any complex number to polar form:

1. Find the magnitude by phasor addition of the *j* term and real term.
2. Find the angle whose tangent is the *j* term divided by the real term. As examples:

$$2 + j4 = 4.47\angle 63°$$

$$4 + j2 = 4.47\angle 26.5°$$

$$8 + j6 = 10\angle 37°$$

$$8 - j6 = 10\angle -37°$$

$$4 + j4 = 5.66\angle 45°$$

$$4 - j4 = 5.66\angle -45°$$

These examples are the same as those given before for finding the magnitude and phase angle of a complex number.

The magnitude in polar form must be more than either term in rectangular form, but less than the arithmetic sum of the two terms. For instance, in 8 + j6 = $10\angle 37°$ the magnitude of 10 is more than 8 or 6 but less than their sum of 14.

Applied to ac circuits with resistance for the real term and reactance for the *j* term, then, the polar form of a complex number states the resultant impedance and its phase angle. Note the following cases for an impedance where either the resistance or reactance is reduced to zero.

$$0 + j5 = 5\angle 90°$$

$$0 - j5 = 5\angle -90°$$

$$5 + j0 = 5\angle 0°$$

The polar form is much more convenient for multiplying or dividing complex numbers. The reason is that multiplication in polar form is reduced to addition of the angles, and the angles are just subtracted for division in polar form. The following rules apply.

For Multiplication

Multiply the magnitudes but add the angles algebraically:

$$24\angle 40° x 2\angle 30° = 48\angle +70°$$
$$24\angle 40° x(-2\angle 30°) = -48\angle +70°$$
$$12\angle -20° x 3\angle -50° = 36\angle -70°$$
$$12\angle -20° 4\angle 5° = 48\angle -15°$$

When you multiply by a real number, just multiply the magnitudes:

$$4 x 2\angle 30° = 8\angle 30°$$
$$4 x 2\angle -30° = 8\angle -30°$$
$$-4 x 2\angle 30° = -8\angle 30°$$
$$-4 x(-2\angle 30°) = 8\angle 30°$$

This rule follows from the fact that a real number has an angle of 0°. When you add 0° to any angle, the sum equals the same angle.

For Division

Divide the magnitudes but subtract the angles algebraically:

$$24\angle 40° \div 2\angle 30° = 12\angle 40° - 30°$$
$$= 12\angle 10°$$
$$12\angle 20° \div 3\angle 50° = 4\angle 20° - 50°$$
$$= 4\angle - 30°$$
$$12\angle -20° \div 4\angle 50° = 3\angle - 20° - 50°$$
$$= 3\angle - 70°$$

To divide by a real number, just divide the magnitudes:

$$12\angle 30° \div 2 = 6\angle 30°$$
$$12\angle -30° \div 2 = 6\angle -30°$$

This rule is also a special case that follows from the fact that a real number has a phase angle of 0°. When you subtract 0° from any angle, the remainder equals the same angle.

For the opposite case, however, when you divide a real number by a complex number, the angle of the denominator changes its sign in the answer in the numerator. This rule still follows the procedure of subtracting angles for division, since a real number has a phase angle of 0°. As examples,

$$\frac{10}{5\angle 30°} = \frac{10\angle 0°}{5\angle 30°}$$
$$= 2\angle 0° - 30° = 2\angle -30°$$
$$\frac{10}{5\angle -30°} = \frac{10\angle 0°}{5\angle -30°}$$
$$= 2\angle 0° - (-30°) = 2\angle +30°$$

Stated another way, we can say that the reciprocal of an angle is the same angle but with opposite sign. Note that this operation is similar to working with powers of 10. Angles and powers of 10 follow the general rules of exponents.

Practice Problems — Section 8

a. $6\angle 20° x 2\angle 30° = ?$

b. $6\angle 20° \div 2\angle 30° = ?$

Complex number in polar form are convenient for multiplication and division, but they cannot be added or subtracted. The reason is that changing the angle corresponds to the operation of multiplying or dividing. When complex numbers in polar form are to be added or subtracted, therefore, they must be converted back into rectangular form.

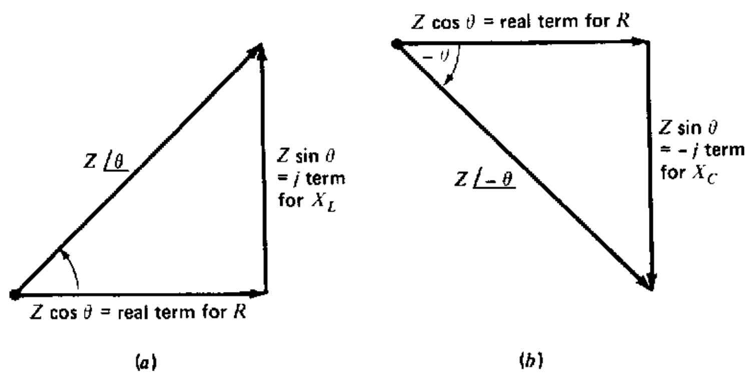


Figure 9
Converting Polar Form of $Z\angle\theta$ to Rectangular Form of $R \pm jX$
(a) Positive Angle θ in First Quadrant has + *j* Term
(b) Negative Angle $-\theta$ in Fourth Quadrant has *-j Term*

Consider the impedance $Z\angle\theta$ in polar form. Its value is the hypotenuse of a right triangle with sides formed by the real term and *j* term in rectangular coordinates. See Figure 9. Therefore, the polar form can be converted to rectangular form by finding the horizontal and vertical sides of the right triangle. Specifically:

$$\text{Real term for R} = Z\cos\theta$$

$$j \text{ term for X} = Z\sin\theta$$

In Figure 9a, assume that $Z\angle\theta$ in polar form is $5\angle 37°$. The sine of 37° is 0.6 and its cosine is 0.8.

To convert to rectangular form:

$$R = Z\cos\theta = 5 \times 0.8 = 4$$

$$X = Z\sin\theta = 5 \times 0.6 = 3$$

Therefore,

$$5\angle 37° = 4 + j3$$

This example is the same as the illustration in Figure 8. The + sign for the *j* term means it is X_L, not X_C.

In Figure 9b, the values are the same, but the *j* term is negative when θ is negative. The negative angle has a negative *j* term because the opposite side is in the fourth quadrant, where the sine is negative. However, the real term is still positive because the cosine is positive.

Note that R for cos θ is the horizontal phasor, which is an adjacent side of the angle. The X for sine θ is the vertical phasor, which is opposite the angle. The +X is X_L; the -X is X_C. You can ignore the sign of θ in calculating sin θ and cos θ because the values are the same up to +90° or down to -90°.

These rules apply for angles in the first or fourth quadrant, from 0 to 90° or from 0 to -90°. As examples:

$$14.14\angle 45° = 10 + j10$$
$$14.14\angle -45° = 10 - j10$$
$$10\angle 90° = 0 + j10$$
$$10\angle -90° = 0 - j10$$
$$100\angle 30° = 86.6 + j50$$
$$100\angle -30° = 86.6 - j50$$
$$100\angle 60° = 50 + j86.6$$
$$100\angle -60° = 50 + j86.6$$

When going from one form to the other, keep in mind whether the angle is smaller or greater than 45° and if the *j* term is smaller or larger than the real term.

For angles between 0 and 45°, the opposite side, which is the *j* term, must be smaller than the real term. For angles between 45 and 90°, the *j* term must be larger than the real term.

To summarize how complex numbers are used in ac circuits in rectangular and polar form:

1. For addition or subtraction, complex number must be in rectangular form. This procedure applies to the addition of impedances in a series circuit. If the series impedances are in rectangular form, just combine all the real terms and *j* terms separately. If the series impedances are in polar form, they must be converted to rectangular form to be added.

2. For multiplication and division, complex numbers are generally used in polar form because the calculations are faster. If the complex number is in rectangular form, convert to polar form. With the complex number available in both forms then you can quickly add or subtract in rectangular form and multiply or divide in polar form. Sample problems showing how to apply these methods in the analysis of ac circuits are illustrated in the following sections.

Practice Problems — Section 9

Convert to rectangular form.

a. $14.14\angle 45°$.

b. $14.14\angle -45°$.

Complex Numbers in Series AC Circuits

Refer to the diagram in Figure 10 on the next page. Although a circuit like this with only series resistances and reactances can be solved just by phasors, the complex numbers show more details of the phase angles.

Z_T in Rectangular Form

The total Z_T in Figure 10a is the sum of the impedances:

$$Z_T = 2 + j4 + 4 - j12$$

$$= 6 - j8$$

The total series impedance then is 6 - j8. Actually, this amounts to adding all the series resistances for the real term and finding the algebraic sum of all the series reactances for the *j* term.

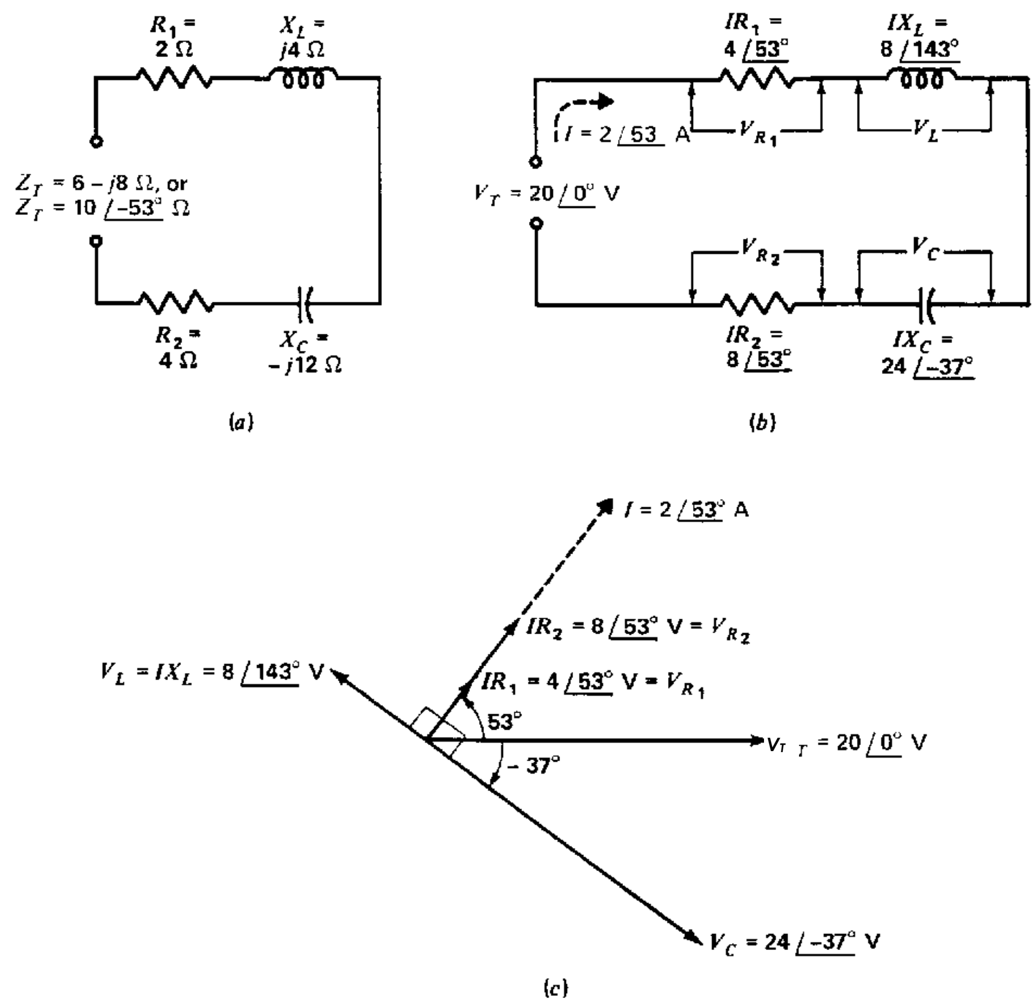


Figure 10
Complex Numbers Applied to Series AC Circuits
(a) Circuit with Series Impedances (b) Current and Voltages
(c) Phasor Diagram of Current and Voltages

Z_T in Polar Form

We can convert Z_T from rectangular to polar form as follows:

$$
\begin{aligned}
Z_T &= 6 - j8 \\
&= \sqrt{36+64}\angle \arctan - 8/6 \\
&= \sqrt{100}\angle \arctan - 1.33 \\
Z_T &= 10\angle 53°\Omega
\end{aligned}
$$

The angle of -53° for Z_T means this is the phase angle of the circuit. Or the applied voltage and the current are 53° out of phase.

Calculating I

The reason for the polar form is to divide Z_T into the applied voltage V_T to calculate the current I. See Figure 10b. Note that the V_T of 20 V is a real number without any *j* term. Therefore, the applied voltage is $20\angle 0°$. This angle of 0° for V_T makes it the reference phase for the following calculations. We can find the current as

$$
\begin{aligned}
I &= \frac{V_T}{Z_T} = \frac{20\angle 0°}{10\angle -53°} \\
&= 2\underline{\angle 0° - (-53°)} \\
I &= 2\angle 53° A
\end{aligned}
$$

Note that Z_T has the negative angle of -53° but the sign changes to +53° for I because of the division into a quantity with the angle of 0°. In general, the reciprocal of an angle in polar form is the same angle with opposite sign.

Phase Angle of the Circuit

The fact that I has the angle of +53° means it leads V_T. The positive angle for I shows the series circuit is capacitive, with leading current. This angle is more than 45° because the net reactance is more than the total resistance, resulting in a tangent function greater than 1.

Finding Each IR Drop

To calculate the voltage drops around the circuit, each resistance or reactance can be multiplied by I:

$$
\begin{aligned}
V_{R_1} &= IR_1 = 2\angle 53° x 2\angle 0° = 4\angle 53° V \\
V_L &= IX_L = 2\angle 53° x 4\angle 90° = 8\angle 143° V \\
V_C &= IX_C = 2\angle 53° x 12\angle -90° = 24\angle -37° V \\
V_{R_2} &= IR_2 = 2\angle 53° x 4\angle 0° = 8\angle 53° V
\end{aligned}
$$

Phase of Each Voltage

The phasors for these voltages are in Figure 10c. They show the phase angles using the applied voltage V_T as the zero reference phase.

The angle of 53° for VR_1 and VR_2 shows that the voltage across a resistance has the same phase as I. These voltages lead V_T by 53° because of the leading current.

For V_C, its angle of -37° means it lags the generator voltage V_T by this much. However, this voltage across X_C still lags the current by 90°, which is the difference between 53° and -37°.

The angle of 143° for V_L in the second quadrant is still 90° leading the current at 53°, as 143° - 53° = 90°. With respect to the generator voltage V_T, though, the phase angle of V_L is 143°.

V_T Equals the Phasor Sum of the Series Voltage Drops

If we want to add the voltage drops around the circuit to see if they equal the applied voltage, each V must be converted to rectangular form. Then these values can be added. In rectangular form then the individual voltages are

$$\begin{aligned}
V_{R_1} &= 4\angle 53^\circ &&= 2.408 + j3.196\ \text{V} \\
V_L &= 8\angle 143^\circ &&= -6.392 + j4.816\ \text{V} \\
V_C &= 24\angle -37^\circ &&= 19.176 - j14.448\ \text{V} \\
V_{R_2} &= 8\angle 53^\circ &&= 4.816 + j6.392\ \text{V} \\
\text{Total V} & &&= 20.008 - j0.044\text{V}
\end{aligned}$$

or converting to polar form,

$$V_T = 20\angle 0^\circ\ V \ \text{ approximately}$$

Note that for $8\angle 143^\circ$ in the second quadrant, the cosine is negative for a negative real term but the sine is positive for a positive *j* term.

Practice Problems — Section 10

Refer to Figure 10.

a. What is the phase of I to V_T?

b. What is the phase of V_L to V_T?

c. What is the phase of V_L to V_R?

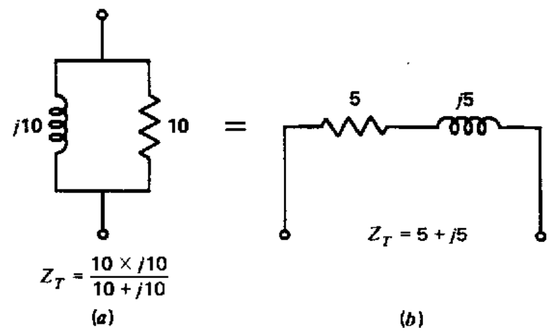


Figure 11
Complex Numbers Used for Parallel AC Circuit to Convert a Parallel Bank to an Equivalent Series Impedance

A useful application here is converting a parallel circuit to an equivalent series circuit. See Figure 11, with a 10-Ω X_L in parallel with a 10-Ω R. In complex notation, R is 10 + j0 while X_L is 0 + j10. Their combined parallel impedance Z_T equals the product over the sum. For Figure 11a, then:

$$Z_T = \frac{(10 + j0)x(0 + j10)}{(10 + j0) + (0 + j10)}$$

$$= \frac{10xj10}{10 + j10}$$

$$Z_T = \frac{j100}{10 + j10}$$

Converting to polar form for division,

$$Z_T = \frac{j100}{10 + j10} = \frac{100\angle 90°}{14.14\angle 45°} = 7.07\angle 45°$$

Converting to Z_T of $7.07\angle 45°$ into rectangular form to see its resistive and reactive components,

Real term = 7.07 cos 45°

= 7.07 x 0.707 = 5

j term = 7.07 sin 45°

$$= 7.07 \times 0.707 = 5$$

Therefore,

$$Z_T = 7.07\angle 45° \text{ in polar form}$$

$$Z_T = 5 + j5 \quad \text{in rectangular form}$$

The rectangular form of Z_T means that 5-Ω R in series with 5-Ω X_L is the equivalent of 10-Ω R in parallel with 10-Ω X_L, as shown in Figure 11b.

Admittance Y and Susceptance B

In parallel circuits, it is usually easier to add branch currents than to combine reciprocal impedances. For this reason, branch conductance G is often used instead of branch resistance, where G = 1/R. Similarly, reciprocal terms can be defined for complex impedances. The two main types are admittance Y, which is the reciprocal of impedance, and susceptance B, which is the reciprocal of reactance. These reciprocals can be summarized as follows:

$$\text{Conductance} = G = \frac{1}{R} \quad S$$

$$\text{Susceptance} = B = \frac{1}{\pm X} \quad S$$

$$\text{Admittance} = Y = \frac{1}{Z} \quad S$$

With R, X, and Z in units of ohms, the reciprocals G, B and Y are in siemens (S) units.

The phase angle for B or Y is the same as current. Therefore, the sign is opposite from the angle of X or Z because of the reciprocal relation. An inductive branch has suceptance -jB, while a capacitive branch has susceptance +jB, with the same angle as branch current.

With parallel branches of conductance and susceptance the total admittance $Y_T = G \pm jB$. For the two branches in Figure 11a, as an example, G is 1/10 or 0.1 and B is also 0.1. In rectangular form.

$$Y_T = 0.1 - j0.1S$$

In polar form,

$$Y_T = 0.14\angle -45^\circ \text{ S}$$

This value for Y_T is the same as I_T with 1 V applied across Z_T of $7.07\angle 45^\circ$ Ω.

As another example, suppose that a parallel circuit has 4 Ω for R in one branch and -j4 Ω for X_C in the other branch. In rectangular form, then, Y_T is 0.25 + j0.25 S. Also, the polar form is $Y_T = 0.35\angle 45^\circ$ S.

Practice Problems — Section 11

a. A Z of 3 + j4 Ω is in parallel with an R of 2 Ω. State Z_T in rectangular form.

b. Do the same as in Prob. a for X_C instead of X_L.

Combining Two Complex Branch Impedances

A common application is a circuit with two branches Z_1 and Z_2, where each is a complex impedance with both reactance and resistance. See Figure 12. A circuit like this can be solved only graphically or by complex numbers. Actually, using complex numbers is the shortest method.

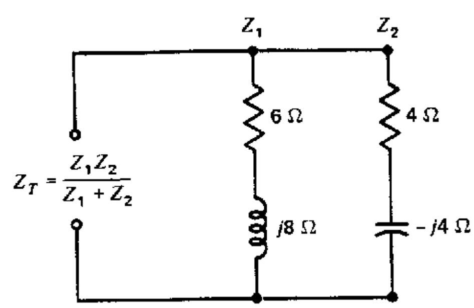


Figure 12
Finding Z_T For Any Two Complex Impedances
Z_1 and Z_2 in Parallel

The procedure here is to find Z_T as the product divided by the sum for Z_1 and Z_2. A good way to start is to state each branch impedance in both rectangular and polar forms. Then Z_1 and Z_2 are ready for addition, multiplication, and division. The solution of this circuit follows:

$$Z_1 = 6 + j8 = 10\angle 53^\circ$$
$$Z_2 = 4 - j4 = 5.66\angle -45^\circ$$

The combined impedance

$$Z_T = \frac{Z_1 x Z_2}{Z_1 + Z_2}$$

Use the polar form of Z_1 and Z_2 to multiply, but add in rectangular form:

$$Z_T = \frac{10\angle 53^\circ x 5.66\angle -45^\circ}{6 + j8 + 4 - j4}$$
$$= \frac{56.6\angle 8^\circ}{10 + j4}$$

Converting the denominator to polar form for easier division,

$$10 + j4 = 10.8\angle 22^\circ$$

Then

$$Z_T = \frac{56.6\angle 8^\circ}{10.8\angle 22^\circ}$$

Therefore

$$Z_T = 5.24\Omega\angle -14^\circ$$

We can convert Z_T into rectangular form. The R component is 5.24 x cos (-14°) or 5.24 x 0.97 = 5.08. Note that cos θ is positive in the first and fourth quadrants. The *j* component equals 5.24 x sin (-14°) or 5.24 x (-0.242) = -1.127. In rectangular form, then,

$$Z_T = 5.08 - j1.27$$

Therefore, this series-parallel circuit combination is equivalent to 5.08 Ω of R in series with 1.27 Ω of X_C. This problem can also be done in rectangular form by rationalizing the fraction for Z_T.

Practice Problems — Section 12

Refer to Figure 12.

a. Add (6 + j8) + (4 - j4) for the sum of Z_1 and Z_2.

b. Multiply for the $10\angle 53° x 5.66\angle -45°$ product of Z_1 and Z_2.

An example with two branches is shown in Figure 13, to find I_T. The branch currents can just be added in rectangular form for the total I_T of parallel branches. This method corresponds to adding series impedances in rectangular form to find Z_T. The rectangular form is necessary for the addition of phasors.

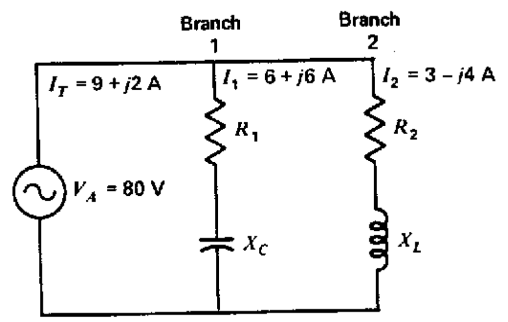


Figure 13
Fining I_T For Two Complex Branch Currents in Parallel

Adding the branch currents in Figure 13,

$$\begin{aligned} I_T &= I_1 + I_2 \\ &= (6 + j6) + (3 - j4) \\ I_T &= 9 + j2 \text{ A} \end{aligned}$$

Note that I_1 has +*j* for the +90° of capacitive current, while I_2 has -*j* for inductive current. These current phasors have the opposite signs from their reactance phasors.

In polar form the I_T of 9 + j2 A is calculated as the phasor sum of the branch currents.

$$\begin{aligned} I_T &= \sqrt{9^2 + 2^2} \\ &= \sqrt{85} = 9.22 \text{ A} \\ \tan\theta &= 2/9 = 0.22 \\ \theta &= 12.53° \end{aligned}$$

Therefore, I_T is 9 + j2 A in rectangular form or $9.22\angle 12.53°$ A in polar form. The complex currents for any number of branches can be added in rectangular form.

a. Find I_T in rectangular form for I_1 of 0 + j2 A and I_2 of 4 + j3 A.

b. Find I_T in rectangular form for I_1 of 6 + j7 A and I_2 of 3 - j9 A.

Parallel Circuit with Three Complex Branches

Because the circuit in Figure 14 has more than two complex impedances in parallel, the method of branch currents is used. There will be several conversions between rectangular and polar form, since addition must be in rectangular form, but division is easier in polar form. The sequence of calculations is:

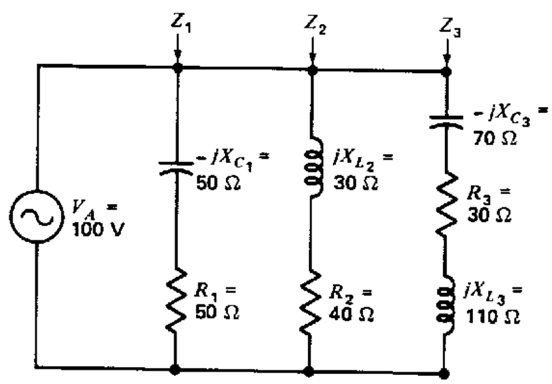


Figure 14
Finding Z_T For Any Three Complex Impedances In Parallel

1. Convert each branch impedance to polar form. This is necessary for dividing into the applied voltage V_A to calculate the individual branch currents. If V_A is not given, any convenient value can be assumed. Note that V_A has a phase angle of 0° because it is the reference.

2. Convert the individual branch currents from polar to rectangular form so that they can be added for the total line current. This step is necessary because the resistive and reactive components must be added separately.

3. Convert the total line current from rectangular to polar form for dividing into the applied voltage to calculate Z_T.
4. The total impedance can remain in polar form with its magnitude and phase angle, or can be converted to rectangular form for its resistive and reactive components.

These steps are used in the following calculations to solve the circuit in Figure 14. All the values are in A, V, or Ω units.

Branch Impedances

Each Z is converted from rectangular form to polar form:

$$Z_1 = 50 - j50 = 70.7\angle -45°$$
$$Z_2 = 40 + j30 = 50\angle +37°$$
$$Z_3 = 30 + j40 = 50\angle +53°$$

Branch Currents

Each I is calculated at V_A divided by Z in polar form:

$$I_1 = \frac{V_A}{Z_1} = \frac{100}{70.7\angle -45°} = 1.414\angle +45° = 1 + j1$$
$$I_2 = \frac{V_A}{Z_2} = \frac{100}{50\angle 37°} = 2.00\angle -37° = 1.6 - j1.2$$
$$I_3 = \frac{V_A}{Z_3} = \frac{100}{50\angle 53°} = 2.00\angle -53° = 1.2 - j1.6$$

The polar form of each I is converted to rectangular form, for addition of the branch currents.

Total Line Current

In rectangular form,

$$I_T = I_1 + I_2 + I_3$$
$$= (1 + j1) + (1.6 - j1.2) + (1.2 - j1.6)$$
$$= 1 + 1.6 + 1.2 + j1 - j1.2 - j1.6$$
$$I_T = 3.8 - j1.8$$

Converting 3.8 - j1.8 into polar form,

$$I_T = 4.2\angle -25.4°$$

Total Impedance

In polar form,

$$Z_T = \frac{V_A}{I_T} = \frac{100}{4.2\angle -25.4^\circ}$$
$$Z_T = 23.8\angle +25.4^\circ$$

Converting $23.8\angle +25.4^\circ$ into rectangular form,

$$Z_T = 21.5 + j10.2$$

Therefore, the complex ac circuit in Figure 14 is equivalent to the combination of 21.5 Ω of R in series with 10.2 Ω of X_L.

This problem can also be done by combining Z_1 and Z_2 in parallel as $Z_1Z_2/(Z_1 + Z_2)$. Then combine this value with Z_3 in parallel to find the total Z_T of the three branches.

Practice Problems — Section 14

Refer to Figure 14.

a. State Z_2 in rectangular form for branch 2.

b. State Z_2 in polar form

c. Find I_2.

Summary

1. In complex numbers, resistance R is a real term and reactance is a *j* term. Thus, an 8-Ω R is 8; an 8-Ω X_L is j8; an 8-Ω X_C is -j8. The general form of a complex impedance with series resistance and reactance then is Z = R ± jX, in rectangular form.
2. The same notation can be used for series voltages where $V = V_R \pm jV_X$.
3. For branch currents $I_T = I_R \pm jI_X$, but the reactive branch currents have signed opposite from impedances. Capacitive branch current is jI_C, while inductive branch current is $-jI_L$.
4. The complex branch currents are added in rectangular form for any number of branches to find I_T.
5. To convert from rectangular to polar form: $R \pm jX = Z\angle\theta$. The magnitude of Z is $\sqrt{R^2 + X^2}$. Also, θ is the angle with tan = X/R.
6. To convert to polar to rectangular form, $Z\angle\theta = R \pm jX$, where R is Z cos θ and the *j* term is Z sin θ. A positive angle has a positive *j* term; a negative angle has a negative *j* term. Also, the angle is more than 45° for a *j* term larger than the real term; the angle is less than 45° for a *j* term smaller than the real term.
7. The rectangular form must be used for addition or subtraction of complex numbers.
8. The polar form is usually more convenient in multiplying and dividing complex numbers. For multiplication, multiply the magnitudes and add the angles; for division, divide the magnitudes and subtract the angles.
9. To find the total impedance Z_T of a series circuit, and all the resistances for the real term and find the algebraic sum of the reactances for the *j* term. The result is $Z_T = R \pm jX$. Then convert Z_T to polar form for dividing into the applied voltage to calculate the current.
10. To find the total impedance Z_T of two complex branch impedances Z_1 and Z_2 in parallel, Z_T can be calculated as $Z_1Z_2/(Z_1 + Z_2)$.

Match the values in the column at the left with those at the right.

1.	24 + j5 + 16 + j10	a.	$14\angle 50°$
2.	24 - j5 + 16 - j10	b.	$7\angle 6°$
3.	j12 x 4	c.	1200 - j800 Ω
4.	j12 x j4	d.	40 + j15
5.	j12 ÷ j3	e.	90 + j60 V
6.	(4 + j2) x (4 - j2)	f.	$45\angle 42°$
7.	1200 Ω of R + 800 Ω of X_C	g.	$24\angle -45°$
8.	5 A of I_R + 7 A of I_C	h.	4
9.	90 V of V_R + 60 V of V_L	I.	j48
10.	$14\angle 28° x \angle 22°$	*j*.	-48
11.	$14\angle 28° \div 2\angle 22°$	k.	5 + j7 A
12.	$15\angle 42° x 3\angle 0°$	l.	20
13.	$6\angle -75° x 4\angle 30°$	m.	40 - j15

1. Give the mathematical operator for the angles of 0°, 90°, 180°, 270°, and 360°.
2. Define the sine, cosine, and tangent functions of an angle.
3. How are mathematical operators similar for logarithms, exponents, and angles?
4. Compare the following combinations: resistance R and conductance G, reactance X and susceptance B, impedance Z and admittance Y.
5. What are the units for admittance Y and susceptance B?
6. Why do Z_T and I_1 for a circuit have angles with opposite signs?

Problems

1. State Z in rectangular form for the following series circuits: (a) 4-Ω R and 3-Ω X_C; (b) 4-Ω R and 3-Ω X_L; (c) 3-Ω R and 6-Ω X_L; (d) 3-Ω R and 3-Ω X_C.
2. Draw the schematic diagrams for the impedances in Prob. 1.
3. Convert the following impedances to polar form: (a) 4 - j3; (b) 4 + j3; (c) 3 + *j*; (d) 3 - j3.
4. Convert the following impedances to rectangular form: (a) $5\angle -27°$; (b) $5\angle 27°$; (c) $6.71\angle 63.4°$; (d) $4.24\angle -45°$.
5. Find the total Z_T in rectangular form for the following three series impedances: (a) $12\angle 10°$; (b) $25\angle 15°$; (c) $34\angle 26°$.
6. Multiply the following, in polar form: (a) $45\angle 24° x 10\angle 54°$; (b) $45\angle -24° x 10\angle 54°$; (c) $18\angle -64° x 4\angle 14°$; (d) $18\angle -64° x 4\angle -14°$.
7. Divide the following, in polar form: (a) $45\angle 24° \div 10\angle 10°$; (b) $45\angle 24° \div 10\angle -10\hat{}$; (c) $500\angle -72° \div 5\angle 12°$; (d) $500\angle -72° \div 5\angle -12°$.
8. Match the four phasor diagrams in Figure 4a, b, c, and d with the four circuits in Figs. 5 and 6.
9. Find Z_T in polar form for the series circuit in Figure 7a.
10. Find Z_T in polar form for the series-parallel circuit in Figure 7c.
11. Solve the circuit in Figure 12 to find Z_T in rectangular form by rationalization.
12. Solve the circuit in Figure 12 to find Z_T in polar form, using the method of branch currents. Assume an applied voltage of 56.6 V.
13. Show the equivalent series circuit of Figure 12.
14. Solve the circuit in Figure 14 to find Z_T in polar form, without using branch currents. (Find the Z of two branches in parallel; then combine this Z with the third branch Z.)
15. Show the equivalent series circuit of Figure 14.

16. Refer to Figure 13, (a) Find Z_1 and Z_2 for the two branch currents given. (b) Calculate the values needed for R_1, R_2, X_C, and X_L for these impedances. (c) What are the L and C values for a frequency of 60 Hz?

17. Solve the series ac circuit in Figure 8 in the previous chapter by the use of complex numbers. Find $Z\angle\theta$, $I\angle\theta$, and each $V\angle\theta$. Prove that the sum of the complex voltage drops around the circuit equals the applied voltage V_T. Make a phasor diagram showing all phase angles with respect to V_T.

18. The following components are in series: L = 100 μH, C = 20 pF, R = 2000 Ω. At the frequency of 2 MHz calculate X_L, X_C, Z_T, I, θ, V_R, V_L, and V_C. The applied V_T = 8 V.

19. Solve the same circuit as in Prob., 18 for the frequency of 4 MHz. Give three effects of the higher frequency.

20. In Figure 15, show that Z_T = 4.8 Ω and θ = 36.9° by (a) the method of branch currents; (b) calculating Z_T as $Z_1Z_2/(Z_1 + Z_2)$.

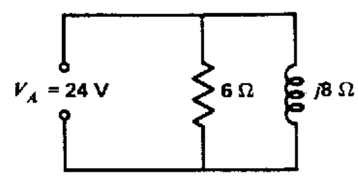


Figure 15

21. In Figure 16, find $Z_T\angle\theta$ by calculating Z_{bc} of the parallel bank and combining with the series Z_{ab}.

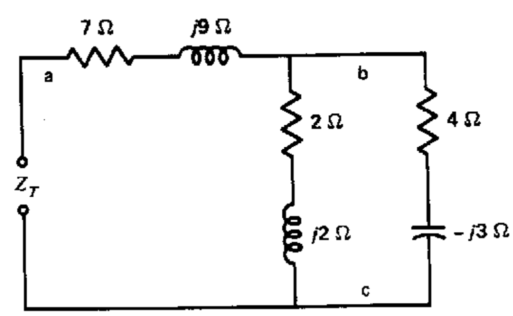


Figure 16

Answers to Practice Problems

Section 1	a. 0°
	b. 180°
Section 2	a. 90°
	b. -90 or 270°
Section 3	a. T
	b. T
Section 4	a. j3 kΩ
	b. -j5 mA
Section 5	a. 4 + j7
	b. 0 - j7
Section 6	a. 5 + j7
	b. 4 + j6
Section 7	a. 14.14Ω
	b. 45°
Section 8	a. $12\angle 50°$
	b. $3\angle -10°$
Section 9	a. 10 + j10
	b. 10 - j10
Section 10	a. 53°
	b. 143°
	c. 90°
Section 11	a. (6 + j8)/(5 + j4)
	b. (6 - j8)/(5 - j4)
Section 12	a. 10 + j4
	b. $56.6\angle 8°$
Section 13	a. 4 + j5 A
	b. 9 - j2 A
Section 14	a. 40 + j30
	b. $50\angle 37°\ \Omega$
	c. $2\angle -37°\ A$

1. (a) 4 - j3
   (b) 4 + j3
   (c) 3 + j6
   (d) 3 - j3
3. (a) $5\angle -37°$
   (b) $5\angle 37°$
   (c) $3.18\angle 18.5°$
   (D) $4.25\angle -45°$
5. Z_T = 65.36 + J23.48
7. (A) $4.5\angle 14°$
   (b) $4.5\angle 34°$
   (c) $100\angle -84°$
   (d) $100\angle -60°$
9. Z_T = $12.65\angle 18.5°$
11. Z_T = $5.25\angle -14.7°$
13. R = 5.08 Ω
    X_C = 1.27 Ω
15. R = 21.4 Ω
    X_L = 10.2 Ω
17. Z_T = $50\angle -37°$ = 40 - j30 Ω
    I = $2\angle 37°$ = 1.6 + j1.2 A
    V_R = $80\angle 37°$ = 64 + j48V
    V_L = $120\angle 127°$ = -72 + j96 V
    V_C = $180\angle -53°$ = 108 - j144 V
19. Z_T = 2.07 kΩ $\angle 14.6°$ kΩ
    I = 3.88 mA $\angle -14.6°$ mA
21. Z_T = $13.4\angle 46.5°$